

Name: Key

**FO-2133 Forest Measurement  
Exam 1 - 2008**

Given the partial computations below,

1. fill in the missing data elements (45 points)
2. complete the regression computations  
for the model:  $DBH_{ob} = b_0 + b_1 (DBH_{ib}) + \epsilon_i$  (20 points)

| Tree#       | DBH <sub>current</sub> | 1X-Bark | DBH <sub>ib,current</sub> | 1X-RG10 | DBH <sub>ib,past</sub> | DBH <sub>ob,past</sub> | DBH Growth |
|-------------|------------------------|---------|---------------------------|---------|------------------------|------------------------|------------|
| 1           | 5.9                    | 0.8     | 4.3                       | 0.20    | 3.9                    | 5.5                    | 0.4        |
| 2           | 6.9                    | 0.9     | 5.1                       | 0.23    | 4.6                    | 6.4                    | 0.5        |
| 3           | 7.5                    | 1.0     | 5.5                       | 0.25    | 5.0                    | 6.8                    | 0.7        |
| 4           | 8.0                    | 1.1     | 5.8                       | 0.23    | 5.3                    | 7.2                    | 0.8        |
| 5           | 8.8                    | 1.0     | 6.8                       | 0.30    | 6.2                    | 8.3                    | 0.5        |
| 6           | 9.4                    | 1.1     | 7.2                       | 0.32    | 6.6                    | 8.7                    | 0.7        |
| 7           | 10.0                   | 1.1     | 7.8                       | 0.33    | 7.1                    | 9.4                    | 0.6        |
| 8           | 10.9                   | 1.3     | 8.3                       | 0.33    | 7.6                    | 10.0                   | 0.9        |
| 9           | 11.9                   | 1.4     | 9.1                       | 0.40    | 8.3                    | 10.8                   | 1.1        |
| 10          | 12.3                   | 1.4     | 9.5                       | 0.44    | 8.6                    | 11.2                   | 1.1        |
| <b>Sum</b>  | <b>91.6</b>            |         | <b>69.4</b>               |         |                        |                        |            |
| <b>Mean</b> | <b>9.16</b>            |         | <b>6.9</b>                |         |                        |                        |            |

(15)

| Tree#       | X           | Y           | X <sup>2</sup> | Y <sup>2</sup> | XY            | $\hat{Y}$ | $(Y_i - \hat{Y}_i)^2$ |
|-------------|-------------|-------------|----------------|----------------|---------------|-----------|-----------------------|
| 1           | 4.3         | 5.9         | 18.49          | 34.81          | 25.37         | 5.95      | 0.0021                |
| 2           | 5.1         | 6.9         | 26.01          | 47.61          | 35.19         | 6.92      | 0.0004                |
| 3           | 5.5         | 7.5         | 30.25          | 56.25          | 41.25         | 7.41      | 0.0087                |
| 4           | 5.8         | 8.0         | 33.64          | 64.00          | 46.40         | 7.77      | 0.0520                |
| 5           | 6.8         | 8.8         | 46.24          | 77.44          | 59.84         | 8.99      | 0.0359                |
| 6           | 7.2         | 9.4         | 51.84          | 88.36          | 67.68         | 9.48      | 0.0059                |
| 7           | 7.8         | 10.0        | 60.84          | 100.00         | 78.00         | 10.21     | 0.0429                |
| 8           | 8.3         | 10.9        | 68.89          | 118.81         | 90.47         | 10.82     | 0.0071                |
| 9           | 9.1         | 11.9        | 82.81          | 141.61         | 108.29        | 11.79     | 0.0121                |
| 10          | 9.5         | 12.3        | 90.25          | 151.29         | 116.85        | 12.28     | 0.0005                |
| <b>Sum</b>  | <b>69.4</b> | <b>91.6</b> | <b>509.26</b>  | <b>880.18</b>  | <b>669.34</b> |           | <b>0.1676</b>         |
| <b>Mean</b> | <b>6.9</b>  | <b>9.2</b>  |                |                |               |           |                       |

(20)

(10)

|            |               |               |               |
|------------|---------------|---------------|---------------|
| <b>CSS</b> | <b>27.624</b> | <b>41.124</b> | <b>33.636</b> |
|------------|---------------|---------------|---------------|

|           |        |
|-----------|--------|
| $b_1$     | 1.2176 |
| $b_0$     | 0.7096 |
| $r^2$     | 0.9959 |
| $s_{y,x}$ | 0.1447 |

$(5) = \frac{33.636}{27.624}$   
 $(5) = 9.16 - 1.2176(6.9)$   
 $(5) = 1 - \frac{0.1676}{41.124}$   
 $(5) = \sqrt{\frac{0.1676}{41.124}}$

3. The resulting regression equation is:  $DBH_{ob} = 0.7096 + 1.2176 (DBH_{ib})$  (5 points)

4. The quadratic mean DBH of the 10 sample trees is: 9.38 inches (5 points)

$$\overline{DBH}_{qm} = \sqrt{\frac{880.18}{10}} =$$

5. If the 10 sample trees above were measured on a 0.1 acre plot, the estimated mean basal area per acre would be: 48 square feet per acre (5 points)

$$BA/plot = (0.05454)(880.18) = 4.8 \text{ sq ft}$$

$$BA/acre = (4.8 \text{ sq ft}) / 0.1 = 48 \text{ sq ft/acre}$$

6. For the 10 sample trees above, the descriptive statistics for  $DBH_{current}$  are:

- A. Variance  $(s^2) =$  4.569 (5 points)

$$s^2 = \frac{CSS}{n-1} = \frac{41.174}{9}$$

- B. Standard deviation  $(s) =$   $\sqrt{4.569} = 2.138$  (5 points)

- C. Standard error of the mean  $(s_{\bar{x}}) =$   $\sqrt{\frac{4.569}{10}} = 0.676$  (5 points)

- D. Coefficient of variation  $(CV\%) =$   $\frac{2.138}{9.16} \times 100 = 23.3\%$  (5 points)

Formulae:

$$s^2 = \frac{\left[ \sum_{i=1}^n x_i^2 - \frac{(\sum_{i=1}^n x_i)^2}{n} \right]}{(n-1)}$$

$$s = \sqrt{s^2}$$

$$\bar{x} \pm (t_{\alpha, n-1}) s_{\bar{x}}$$

$$CV\% = \left( \frac{s}{\bar{x}} \right) (100)$$

$$s^2 = \frac{\left[ \sum_{i=1}^n x_i^2 - \bar{x} \sum_{i=1}^n x_i \right]}{(n-1)}$$

$$s_{\bar{x}} = \sqrt{\frac{s^2}{n} \left( 1 - \frac{n}{N} \right)}$$

$$SE\% = \left( \frac{t_{\alpha, n-1} s_{\bar{x}}}{\bar{x}} \right) 100\%$$

$$CSS_y: \sum y^2 = \sum Y^2 - \frac{(\sum Y)^2}{n}$$

$$CSS_x: \sum x^2 = \sum X^2 - \frac{(\sum X)^2}{n}$$

$$CSP_{xy}: \sum xy = \sum XY - \frac{\sum X \sum Y}{n}$$

$$b_1 = \frac{\sum xy}{\sum x^2} = \frac{CSP_{xy}}{CSS_x}$$

$$b_0 = \bar{Y} - b_1 \bar{X}$$

$$TotalSS = CSS_y = \sum y^2$$

$$RegSS = \frac{(CSP_{xy})^2}{CSS_x} = \frac{(\sum xy)^2}{\sum x^2} = b_1 \sum xy$$

$$ErrorSS = \sum (Y - \hat{Y})^2$$

$$I^2 = \left( 1 - \frac{ErrorSS}{TotalSS} \right)$$

$$S_{y.x} = \sqrt{\frac{\sum (Y - \hat{Y})^2}{n-2}} = \sqrt{\frac{ErrorSS}{n-2}}$$