FO-2213 Forest Measurements

Topic 19: Double-Point Sampling and Volume:Basal Area Regression

Concept of Double-Point Sampling:

The prism or angle gauge selects trees with probability proportional to size (PPS).

Each tree selected (i.e. that is larger than the critical angle) represents BAF square feet of basal area per acre.

Tree volume is highly correlated to tree basal area; thus, if we can sample basal area, we can then relate/predict volume from basal area estimates.

Bsal area is cheap and efficient to sample (i.e. count trees by species and product class) with a prism/angle gauge. Volume (i.e. measure dbh and height and assign volume) is more difficult and time consuming to measure/determine.

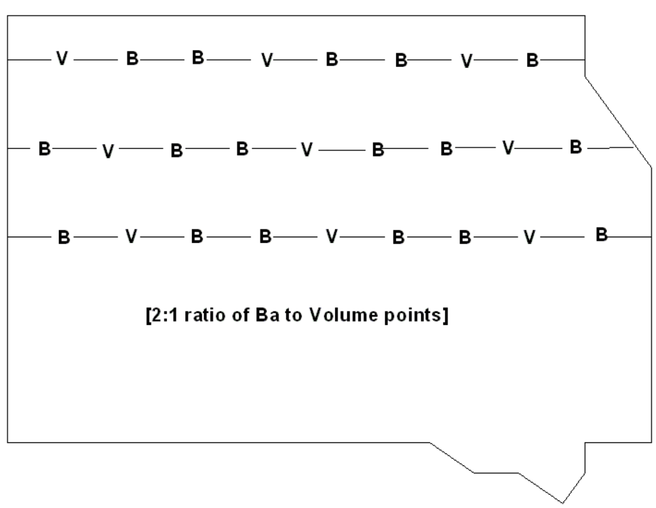
Double-sampling procedure allows us to measure basal area on all sample points and volume on a sub-sample of the basal area points. All point are basal area or count points and the sub-sample are the volume-points that are used to relate volume to basal area.

Field Design of a Double-Sample:

<u>Number of sample points</u> to meet desired precision level is determined for an infinite population, because there are an infinite number of possible sample points.

<u>Distribution of sample points</u> is either systematic or random allocation.

<u>All points are Count or Phase 1 points</u>; volume or Phase 2 points are a subset of the total.
<u>On Count (BA)-Points (Phase 1)</u> only numbers of trees by species-product class are tallied.
<u>On Volume-Points (Phase 2)</u> tree volume is tallied/compute by means of dbh and height by species-product class.



Computations:

1. For the volume points, compute basal area and volume on a per acre basis for each desired species-product class for each point.

A. Compute a linear regression equation for the model: $Volume = b_0 + b_1 \ (Basal\ Area)$

where Y = volume per acre on each point, X = basal area per acre on each point

B. Compute the mean ba/ac for the volume points: $ba / ac = \frac{\sum (trees_i)\ BAF}{No.\ points}$

2. For the count points (all points), compute the overall mean BA/ac: $BA / ac = \frac{\sum (trees_i)\ BAF}{No.\ points}$

3. Compute the linear regression estimate of mean volume per acre on the volume points:

$$\overline{Y}_{lr} = \overline{y}_2 + b_1 \left[BA_1 - ba_2 \right]$$

where:

$\overline{Y}_{lr}$ = adjusted linear regression estimate of volume per acre

$\overline{y}_2$ = mean volume per acre on n_2 volume points (i.e. Phase 2)

b_1 = slope coefficient of regression equation for volume and ba.

BA_1 = Basal Area per acre on n_1 count points (i.e. Phase 1)

ba_2 = basal area per acre on n_2 volume points (i.e. Phase 2)

with fit statistics:

$$S_{yx} = \sqrt{\frac{\sum (Y - \hat{Y})^2}{n_2 - 2}} = \sqrt{\frac{ESS}{n_2 - 2}}$$

$$S_{\overline{y}_{lr}} = \left\{ S^2_{Y.X} \left[\frac{1}{n_2} + \frac{\left(\overline{X}_1 - \overline{X}_2\right)^2}{(n_2 - 1) S^2_X} \right] + \frac{S^2_Y - S^2_{Y.X}}{n_1} \right\}^{\frac{1}{2}}$$

where: $S^2_x = \frac{SS_x}{(n_2 - 1)}$

$S^2_y = \frac{SS_y}{(n_2 - 1)}$

$S^2_{xy} = \frac{SS_{xy}}{(n_2 - 1)}$

Confidence interval: $\overline{Y}_{lr} \pm t_{n-2,\alpha}\ S_{\overline{y}_{lr}}$

Computation Example:

BAF = 10; n_1=15, n_2 =5

Height sub-sample: $\ln(H)= 4.57 - 3.6114\ (dbh^{-1})$

Volume function: $Doyle_{6''} = -57.66 + 0.01241(D^2H)$

$PACF = BAF/(0.005454D^2)$

Point#	DBH	Trees		Hgt	Vol	PACF	Vol/ac	BA	Vol		Y^2	X^2	XY	$(Y-\hat{Y})^2$
1	10	2		67.3	25.8	18.34	947.3							
	11	1		69.5	46.7	15.15	708.3							
	13	3		73.1	95.7	10.85	3115.1							
	15	3		75.9	154.2	8.15	3770.5	90	8541.2		72,952,923	8100	768712.4	190
2		8												
3		9												
4	11	3		69.5	46.7	15.15	2124.8							
	9	2		64.6	7.3	22.64	331.0							
	12	2		71.5	70.0	12.73	1783.4	70	4239.1		17,969,722	4900	296735	322,095
5		7												
6		9												
7	12	3		71.5	70.0	12.73	2675.0							
	15	3		75.9	154.2	8.15	3770.5							
	16	4		77.0	187.1	7.16	5359.7	100	11805.3		139,365,317	10000	1180531	2,009,035
8		8												
9		10												
10	12	2		71.5	70.0	12.73	1783.4							
	13	3		73.1	95.7	10.85	3115.1							
	14	4		74.6	123.8	9.35	4631.6							
	16	2		77.0	187.1	7.16	2679.9	110	12209.9		149,080,914	12100	1343086	1,480
11		10												
12		9												
13	12	3		71.5	70.0	12.73	2675.0							
	13	3		73.1	95.7	10.85	3115.1							
	14	3		74.6	123.8	9.35	3473.7							
	16	3		77.0	187.1	7.16	4019.8	120	13283.6		176,453,830	14400	1594031	680,920
14		10												
15		11												
							Total	490	50,079	**Sum**	555,822,706	49,500	5,183,095	3,013,719
	Sum	140					**Mean**	**98**	**10,016**					
	BA	**93.33**								**CSS**	54,239,653	1480	275344.1	
										b_1	186.04			
										b_o	-8,216			
										r^2	0.94			
										$S_{y.x}$	1,002			

Linear Regression Volume Estimate:

$$\overline{Y}_{lr} = \overline{y}_2 + b_1\left[BA_1 - ba_2\right] \qquad \overline{Y}_{lr} = 10{,}016 + 186.04\left[93.33 - 98.0\right] \qquad \overline{Y}_{lr} = 9{,}147\, bd.\,ft\ per\ acre\ Doyle$$

Also, Doyle = -8,216 + 186.04 (93.33) = 9,147 bd. ft/ac; however, this is not a linear regression estimate. It is the mean volume per acre based on the Phase 1 basal area.

Stand and Stock Table:

1. Sort the original computation by DBH for the volume points only.
2. Sum the expanded trees and volume by DBH class and divide by number of points.

DBH	Trees		Hgt	Vol	PACF	Trees/pt	Vol/pt	DBH	Trees/ac	BA/ac	Vol/ac
9	2		64.6	7.3	22.64	45.3	331	9	9.1	4.0	66
10	2		67.3	25.8	18.34	36.7	947	10	7.3	4.0	189
11	1		69.5	46.7	15.15	15.2	708				
11	3		69.5	46.7	15.15	45.5	2,125	11	12.1	8.0	567
12	2		71.5	70.0	12.73	25.5	1,783				
12	3		71.5	70.0	12.73	38.2	2,675				
12	2		71.5	70.0	12.73	25.5	1,783				
12	3		71.5	70.0	12.73	38.2	2,675	12	25.5	20.0	1,783
13	3		73.1	95.7	10.85	32.5	3,115				
13	3		73.1	95.7	10.85	32.5	3,115				
13	3		73.1	95.7	10.85	32.5	3,115	13	19.5	18.0	1,869
14	4		74.6	123.8	9.35	37.4	4,632				
14	3		74.6	123.8	9.35	28.1	3,474	14	13.1	14.0	1,621
15	3		75.9	154.2	8.15	24.4	3,771				
15	3		75.9	154.2	8.15	24.4	3,771	15	9.8	12.0	1,508
16	4		77.0	187.1	7.16	28.6	5,360				
16	2		77.0	187.1	7.16	14.3	2,680				
16	3		77.0	187.1	7.16	21.5	4,020	16	12.9	18.0	2,412
Sum	**49**					546.4	50,079		**109.3**	**98.0**	**10,016**
Mean	**9.8**					**109.3**	**10,016**				

Note: The sum of Stand and Stock table should equal the computations for the volume points.

Allocation of Phase 1 and Phase 2 Plots in Double Sample

$$N_{rs} = \frac{1}{\frac{1}{N} + \left(\frac{AE\%}{t\,cv\%}\right)^2}$$

Phase 1 sample size[1]:

$$\boldsymbol{n_1} = N_{rs}\left[(1 - \rho^2)\sqrt{\left(\frac{c_2}{c_1}\right)\left(\frac{\rho^2}{1 - \rho^2}\right)} + \rho^2\right]$$

Phase 2 sample size[1]:

$$\boldsymbol{n_2} = N_{rs}\left[(1 - \rho^2) + \rho^2\sqrt{\left(\frac{c_1}{c_2}\right)\left(\frac{1 - \rho^2}{\rho^2}\right)}\right]$$

where ρ^2 is coefficient of determination, c_1 and c_2 are costs of Phase 1 and Phase 2 samples.

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[1] Adapted from Johnson, E.W. 2000. **Forest Sampling Desk Reference**. CRC Press, Boca Raton, FL. 985 pp.