

FO-2213 Forest Measurement
Topic 2: **Statistics Review**

Chapter 2

Why is statistics an important topic and skill in forest measurements?

What can we do with statistics?

What is the difference between accuracy and precision?

Variable of interest: x_i

Mean: $\bar{x} = \frac{\sum_{i=1}^n x_i}{n}$ Variance: (definition) $s^2 = \frac{\sum_{i=1}^n (x_i - \bar{x})^2}{(n - 1)}$

Variance: (working) $s^2 = \frac{\left[\sum_{i=1}^n x_i^2 - \frac{(\sum_{i=1}^n x_i)^2}{n} \right]}{(n - 1)}$ Or $s^2 = \frac{\left[\sum_{i=1}^n x_i^2 - \bar{x} \sum_{i=1}^n x_i \right]}{(n - 1)}$

Standard Deviation: $s = \sqrt{s^2}$

Coefficient of Variation: $CV\% = \left(\frac{s}{\bar{x}} \right) (100)$

Standard Error of the Mean: $s_{\bar{x}} = \sqrt{\frac{s^2}{n} \left(1 - \frac{n}{N} \right)}$

where n = number of samples, N = population size

(1- ∞) Confidence Interval: $\bar{x} \pm (t_{\alpha, n-1}) s_{\bar{x}}$

Sampling Error: $SE\% = \left(\frac{t_{\alpha, n-1} s_{\bar{x}}}{\bar{x}} \right) 100\%$

Sample Size: generic $n = \frac{1}{\frac{1}{N} + \left(\frac{AE\%}{t CV\%} \right)^2}$ where $N = \left(\frac{A}{ps} \right)$

Sample Size: infinite popul $n = \frac{t^2 (CV\%)^2}{(AE\%)^2} = \left[\frac{t CV\%}{AE\%} \right]^2$

Sample Size: finite popul $n = \frac{N t^2 (CV\%)^2}{N (AE\%)^2 + t^2 (CV\%)^2}$

Student's t-Table

The Distribution of t

df	Probability = α								
	0.5	0.4	0.3	0.2	0.1	0.05	0.02	0.01	0.001
1	1.000	1.376	1.963	3.078	6.314	12.706	31.821	63.657	636.619
2	0.819	1.061	1.386	1.886	2.920	4.303	6.965	9.925	31.598
3	0.765	0.978	1.250	1.638	2.353	3.182	4.541	5.841	12.941
4	0.741	0.941	1.190	1.533	2.132	2.776	3.747	4.604	8.610
5	0.727	0.920	1.156	1.476	2.015	2.571	3.365	4.032	6.856
6	0.718	0.906	1.134	1.440	1.943	2.447	3.143	3.707	5.959
7	0.711	0.896	1.119	1.415	1.895	2.365	2.998	3.499	5.405
8	0.706	0.889	1.108	1.397	1.860	2.306	2.896	3.355	5.041
9	0.703	0.883	1.100	1.383	1.833	2.262	2.821	3.250	4.781
10	0.700	0.879	1.093	1.372	1.812	2.228	2.764	3.169	4.587
11	0.697	0.876	1.088	1.363	1.769	2.201	2.718	3.106	4.437
12	0.695	0.873	1.083	1.356	1.782	2.179	2.681	3.055	4.318
13	0.694	0.870	1.079	1.350	1.771	2.160	2.650	3.012	4.221
14	0.692	0.868	1.076	1.345	1.761	2.145	2.624	2.977	4.140
15	0.691	0.866	1.074	1.341	1.753	2.131	2.602	2.947	4.073
16	0.690	0.865	1.071	1.337	1.746	2.120	2.583	2.921	4.015
17	0.689	0.863	1.069	1.333	1.740	2.110	2.567	2.898	3.965
18	0.688	0.862	1.067	1.330	1.734	2.101	2.552	2.878	3.922
19	0.688	0.861	1.066	1.328	1.729	2.093	2.539	2.861	3.883
20	0.687	0.860	1.064	1.325	1.725	2.086	2.528	2.845	3.850
21	0.686	0.859	1.063	1.323	1.721	2.080	2.518	2.831	3.819
22	0.686	0.858	1.061	1.321	1.717	2.074	2.508	2.819	3.792
23	0.685	0.858	1.060	1.319	1.714	2.069	2.500	2.807	3.767
24	0.685	0.857	1.059	1.318	1.711	2.064	2.492	2.797	3.745
25	0.684	0.856	1.058	1.316	1.708	2.060	2.485	2.787	3.725
26	0.684	0.856	1.058	1.315	1.706	2.056	2.479	2.779	3.707
27	0.684	0.855	1.057	1.314	1.703	2.052	2.473	2.771	3.690
28	0.683	0.855	1.056	1.313	1.701	2.048	2.467	2.763	3.674
29	0.683	0.854	1.055	1.311	1.699	2.045	2.462	2.756	3.659
30	0.683	0.854	1.055	1.310	1.697	2.042	2.457	2.750	3.646
40	0.681	0.851	1.050	1.303	1.684	2.021	2.423	2.704	3.551
60	0.679	0.848	1.046	1.296	1.671	2.000	2.390	2.660	3.460
120	0.677	0.845	1.041	1.289	1.658	1.980	2.358	2.617	3.373
∞	0.674	0.842	1.036	1.282	1.645	1.960	2.326	2.576	3.291

Sample Statistical Computations: Tree Heights

tree #	Ht		X	X ²
1	82.5		82.5	6806.25
2	84.5		84.5	7140.25
3	86.3		86.3	7447.69
4	86.8		86.8	7534.24
5	86.8		86.8	7534.24
6	87.0		87.0	7569.00
7	87.7		87.7	7691.29
8	88.6		88.6	7849.96
9	90.0		90.0	8100.00
10	90.1		90.1	8118.01
11	90.8		90.8	8244.64
12	91.7		91.7	8408.89
13	92.5		92.5	8556.25
14	92.8		92.8	8611.84
15	93.0		93.0	8649.00
16	93.9		93.9	8817.21
17	94.2		94.2	8873.64
18	94.9		94.9	9006.01
19	96.7		96.7	9350.89
20	98.0		98.0	9604.00
20		Sum	1808.8	163913.3
n		Mean	90.44	
		Median	90.45	
		Variance	17.1278	
		Std. Dev.	4.1386	
		Std. Error	0.9254	
		Half C.I.	1.9369	
		SE%	2.14%	
		L C.I.	88.50	
		U C.I.	92.38	
		CV%	4.58%	

Statistical Example: Plot Volumes (0.1 acre plots)

Plot#		Cu.Ft/plot	X ²		Cu.Ft/acre	X ₂
1		144	20,736		1440	2073600
2		168	28,224		1680	2822400
3		180	32,400		1800	3240000
4		180	32,400		1800	3240000
5		216	46,656		2160	4665600
6		216	46,656		2160	4665600
7		216	46,656		2160	4665600
8		240	57,600		2400	5760000
9		240	57,600		2400	5760000
10		240	57,600		2400	5760000
11		252	63,504		2520	6350400
12		252	63,504		2520	6350400
13		288	82,944		2880	8294400
14		300	90,000		3000	9000000
15		300	90,000		3000	9000000
16		300	90,000		3000	9000000
17		336	112,896		3360	11289600
18		336	112,896		3360	11289600
19		420	176,400		4200	17640000
20		504	254,016		5040	25401600
20	Sum	5,328	1,562,688		53,280	156,268,800
n	Mean	266.4			2664	
	Median	246				
	Variance	7542.57			754257	
	Std. Dev.	86.84796			868.4796	
	Std. Error	19.4198			190.9339	
	Half C.I.	40.6456			399.6246	
	SE%	15.26%			15.00%	
	L C.I.	225.75			2264.38	
	U C.I.	307.05			3063.62	
	CV%	32.60%			32.60%	

Additional Information: 1/10 acre plots taken in a 60 acre stand

1. Recompute the mean volume per acre.

$$\text{Mean volume/ac} = 266.4/.1 = 2,664 \text{ cu.ft per acre}$$

2. Recompute the precision statistics on a per acre basis: $n = 20$, $N = 60/.1 = 600$

$$\text{Std. Error}_{1.0} = s_{\bar{x}} = \sqrt{\frac{868.48}{20} \left(1 - \frac{20}{600}\right)} = 190.9339$$

3. Recompute the sampling error and confidence intervals.
4. Why did the sampling error change and not the coefficient of variation at the per acre basis?