FO-2213 Forest Measurement

Topic 3: **Simple Linear Regression**

Model: $Y = b_0 + b_1X + \epsilon$

where:
- X = the independent variable
- Y = the dependent variable
- b_0 = value of Y when X=0 = the intercept
- b_1= relationship between y and x; “rise over run” = the slope
- ϵ = the error associated with the estimate of Y.

Regression Assumptions:

1. X is measured without error.
2. Y has a constant variance for all levels of X.
3. For each value of X, Y is normally and independently distributed.

Computation:

$$SS_y:\quad \sum y^2 = \sum Y^2 - \frac{(\sum Y)^2}{n}$$ corrected sum of squares of y’s **(**CSS_y)

$$SS_x:\quad \sum x^2 = \sum X^2 - \frac{(\sum X)^2}{n}$$ corrected sum of squares of x’s (CSS_x)

$$SP_{xy}:\quad \sum xy = \sum XY - \frac{\sum X \sum Y}{n}$$ corrected sum of cross-products (CSP_{xy})

$$b_1 = \frac{\sum xy}{\sum x^2} = \frac{SP_{xy}}{SS_x} \qquad\qquad b_1 = \frac{\sum XY - \frac{\sum X \sum Y}{n}}{\sum X^2 - \frac{(\sum X)^2}{n}}$$

$$b_0 = \bar{Y} - b_1\bar{X}$$

Regression Through the Origin: $Y = b_1X + \epsilon$

$$b_1 = \frac{\sum XY}{\sum X^2} = \frac{USP_{xy}}{USS_x}$$ (Using Uncorrected sum of products and squares)

$b_0 = 0$

Precision Statistics:

Total Sum of Squares: (TSS) $SS_y = \sum y^2$

Regression Sum of Squares: (RSS) $\frac{(SP_{xy})^2}{SS_x} = \frac{(\sum xy)^2}{\sum x^2} = b_1 \sum xy$

Error Sum of Squares: (ESS) $ESS = \sum (Y - \hat{Y})^2$

Coefficient of Determination: (r^2) $r^2 = \frac{Regression\ SS}{Total\ SS}$

Index of Fit: (I^2) $I^2 = \left(1 - \frac{Error\ SS}{Total\ SS}\right)$

Standard Error of the Estimate: ($S_{y.x}$) $S_{y.x} = \sqrt{\frac{\sum (Y - \hat{Y})^2}{n - 2}} = \sqrt{\frac{ESS}{n - 2}}$

$$S_{y.x} = \sqrt{\frac{\sum y_i^2 - b_1 \sum xy}{n - 2}} = \sqrt{\frac{ESS}{n - 2}}$$

Total SS = Regression SS + Error SS (SS for regression model)

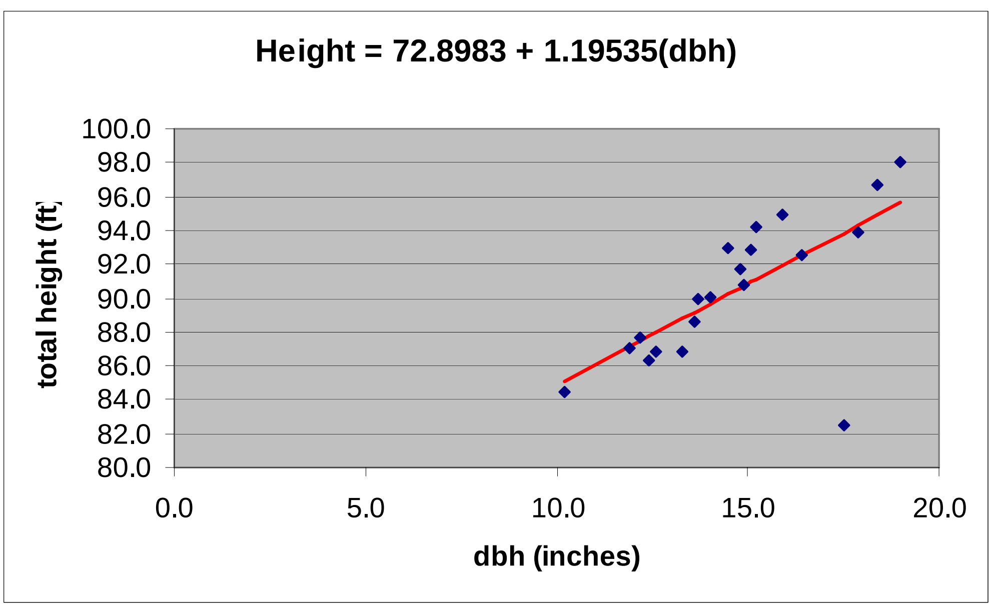
$\frac{\text{Total SS}}{\text{Total SS}} = \frac{\text{Regression SS}}{\text{Total SS}} + \frac{\text{Error SS}}{\text{Total SS}}$ (divide each term by Total SS)

$1 = r^2 + \frac{\text{Error SS}}{\text{Total SS}}$ (r^2 is defined as proportion of SS explained by regression)

thus, $r^2 = 1 - (\text{Error SS} / \text{Total SS})$

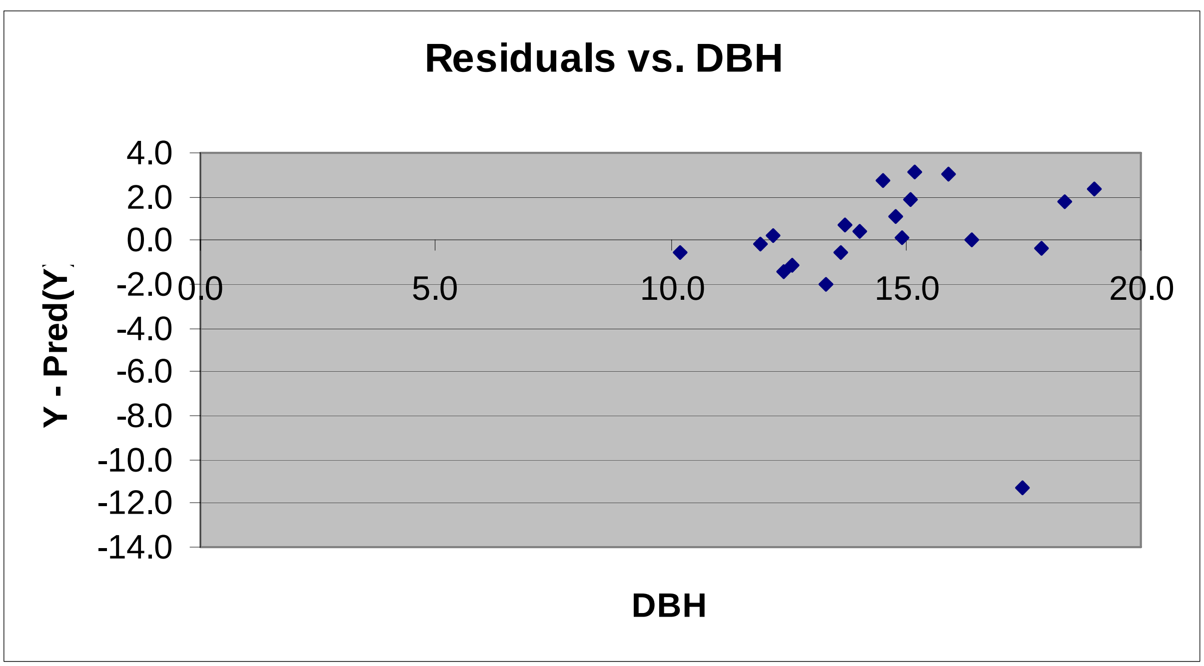
Example 1: Height vs DBH

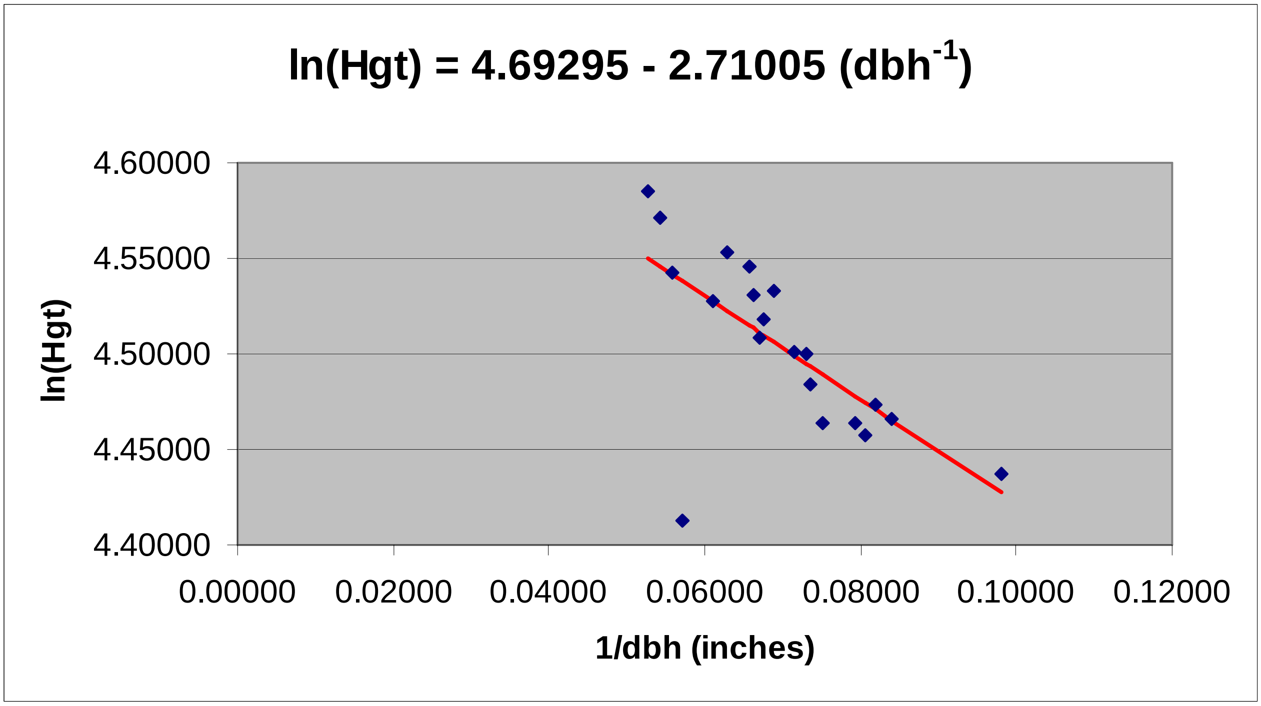
tree #	DBH	Ht		X=DBH	Y=Ht	X^2	Y^2	XY	$(Y-\hat{Y})^2$
1	13.3	86.8		13.3	86.8	176.9	7534.2	1154.4	4.0
2	12.4	86.3		12.4	86.3	153.8	7447.7	1070.1	2.0
3	12.2	87.7		12.2	87.7	148.8	7691.3	1069.9	0.0
4	17.5	82.5		17.5	82.5	306.3	6806.3	1443.8	128.1
5	14.8	91.7		14.8	91.7	219.0	8408.9	1357.2	1.2
6	11.9	87.0		11.9	87.0	141.6	7569.0	1035.3	0.0
7	12.6	86.8		12.6	86.8	158.8	7534.2	1093.7	1.3
8	14.5	93.0		14.5	93.0	210.3	8649.0	1348.5	7.7
9	15.9	94.9		15.9	94.9	252.8	9006.0	1508.9	9.0
10	14.9	90.8		14.9	90.8	222.0	8244.6	1352.9	0.0
11	14.0	90.1		14.0	90.1	196.0	8118.0	1261.4	0.2
12	15.1	92.8		15.1	92.8	228.0	8611.8	1401.3	3.4
13	13.6	88.6		13.6	88.6	185.0	7850.0	1205.0	0.3
14	15.2	94.2		15.2	94.2	231.0	8873.6	1431.8	9.8
15	13.7	90.0		13.7	90.0	187.7	8100.0	1233.0	0.5
16	17.9	93.9		17.9	93.9	320.4	8817.2	1680.8	0.2
17	16.4	92.5		16.4	92.5	269.0	8556.3	1517.0	0.0
18	10.2	84.5		10.2	84.5	104.0	7140.3	861.9	0.3
19	18.4	96.7		18.4	96.7	338.6	9350.9	1779.3	3.3
20	19.0	98.0		19.0	98.0	361.0	9604.0	1862.0	5.7
20			Sum	293.5	1808.8	4410.9	163913.3	26668.2	177.1
n			Mean	14.675	90.44				
			CSS			103.77750	325.42800	124.05000	
			b_1			1.19535			
			b_0			72.89830			
			r^2			0.4557			
			$s_{y.x}$			3.137102			

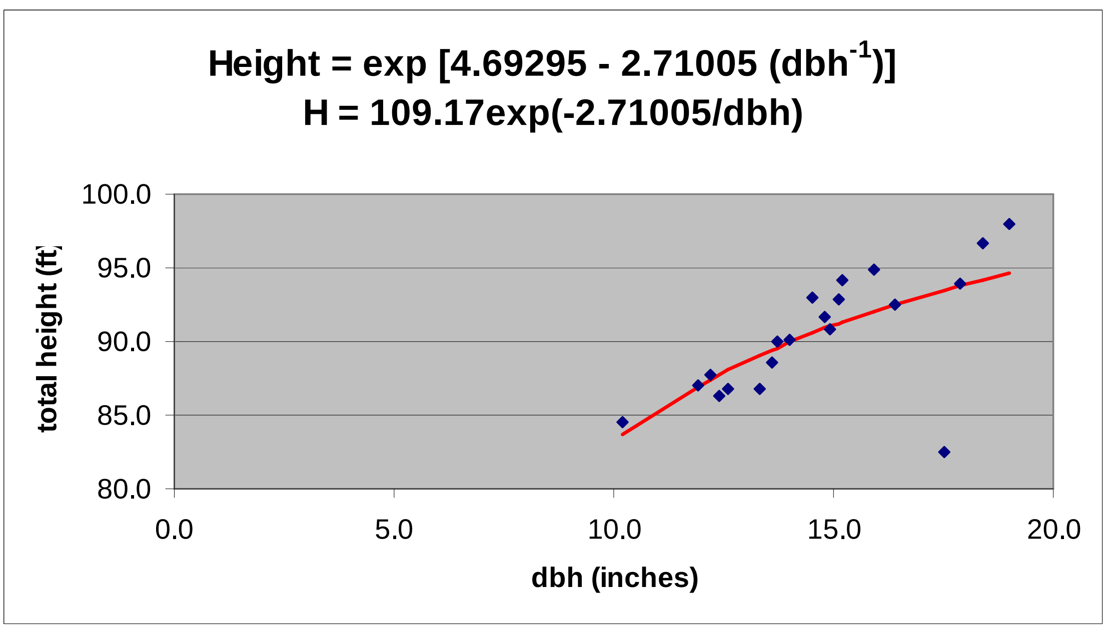


Example 2: LnHgt as a function of dbh^{-1}

tree #	DBH	Ht		$X=dbh^{-1}$	Y=ln(Ht)	X^2	Y^2	XY		y	$\hat{Y}$	$(y-\hat{Y})^2$	y^2
1	13.3	86.8		0.07519	4.46361	0.00565	19.92378	0.33561		86.8	89.04866	5.05649	7534.24
2	12.4	86.3		0.08065	4.45783	0.00650	19.87224	0.35950		86.3	87.74139	2.07760	7447.69
3	12.2	87.7		0.08197	4.47392	0.00672	20.01598	0.36671		87.7	87.42759	0.07421	7691.29
4	17.5	82.5		0.05714	4.41280	0.00327	19.47279	0.25216		82.5	93.51167	121.25679	6806.25
5	14.8	91.7		0.06757	4.51852	0.00457	20.41704	0.30531		91.7	90.90679	0.62918	8408.89
6	11.9	87.0		0.08403	4.46591	0.00706	19.94434	0.37529		87.0	86.93936	0.00368	7569.00
7	12.6	86.8		0.07937	4.46361	0.00630	19.92378	0.35425		86.8	88.04630	1.55327	7534.24
8	14.5	93.0		0.06897	4.53260	0.00476	20.54446	0.31259		93.0	90.56304	5.93878	8649.00
9	15.9	94.9		0.06289	4.55282	0.00396	20.72820	0.28634		94.9	92.06573	8.03308	9006.01
10	14.9	90.8		0.06711	4.50866	0.00450	20.32801	0.30259		90.8	91.01858	0.04778	8244.64
11	14.0	90.1		0.07143	4.50092	0.00510	20.25828	0.32149		90.1	89.96054	0.01945	8118.01
12	15.1	92.8		0.06623	4.53045	0.00439	20.52495	0.30003		92.8	91.23811	2.43950	8611.84
13	13.6	88.6		0.07353	4.48413	0.00541	20.10744	0.32972		88.6	89.44982	0.72219	7849.96
14	15.2	94.2		0.06579	4.54542	0.00433	20.66084	0.29904		94.2	91.34590	8.14588	8873.64
15	13.7	90.0		0.07299	4.49981	0.00533	20.24829	0.32845		90.0	89.58002	0.17638	8100.00
16	17.9	93.9		0.05587	4.54223	0.00312	20.63186	0.25376		93.9	93.83583	0.00412	8817.21
17	16.4	92.5		0.06098	4.52721	0.00372	20.49562	0.27605		92.5	92.54539	0.00206	8556.25
18	10.2	84.5		0.09804	4.43675	0.00961	19.68476	0.43498		84.5	83.70134	0.63786	7140.25
19	18.4	96.7		0.05435	4.57161	0.00295	20.89965	0.24846		96.7	94.22268	6.13713	9350.89
20	19.0	98.0		0.05263	4.58497	0.00277	21.02193	0.24131		98.0	94.66194	11.14266	9604.00
20			Sum	1.39671	90.07378	0.10001	405.70424	6.28365				174.09808	163913
n			Mean	0.06984	4.5036888								
			CSS			0.00247	0.03999	-0.00669					325.428
			b_1			-2.71005							
			b_0			4.69295							
			r^2			0.4650							
			$s_{y.x}$			3.11000							




ln(Hgt) = 4.69295 - 2.71005 (dbh-1)
ln(Hgt)
4.60000
4.55000
4.50000
4.45000
4.40000
0.00000
0.02000
0.04000
0.06000
0.08000
0.10000
0.12000
1/dbh (inches)


Height = exp [4.69295 - 2.71005 (dbh-1)]
H = 109.17exp(-2.71005/dbh)
total height (ft)
100.0
95.0
90.0
85.0
80.0
0.0
5.0
10.0
15.0
20.0
dbh (inches)