

**Topic 5: Regressions of
DBH(ob vs. ib), DBH(growth vs past dbh), and Height-DBH (lnH vs. dbh⁻¹)
“The Basic Tree Exercise”**

Why are these relationships important?

1. For quantifying dbh growth of trees.
2. For establishing the relationship between outside bark (ob) and inside bark (ib) diameters; i.e. bark thickness vs. wood thickness.
3. For predicting dbh growth on sample trees within a forest.
4. For measuring dbh in the field and predicting height which is more difficult to measure.

DBH: diameter, outside-bark, of a tree measured at 4.5 ft above high-side of ground.
DBH is by definition an ob-measurement.

Bark thickness: the thickness of bark at a defined diameter point.

Radial growth: the amount of radial growth (in tree diameter) during a specified time period; i.e. 10 years.

DBH and growth example:

$$\text{DBH (ob)} = \text{DBH (ib)} + 2 * (\text{single bark thickness})$$

$$\text{Thus, DBH (ib)} = \text{DBH (ob)} - 2 * (\text{single bark thickness})$$

$$\text{Likewise, past DBH (ib)} = \text{current DBH (ob)} - 2 * (\text{single bark thick.}) - 2 * (\text{radial growth})$$

$$\text{So, past DBH (ob)} = \text{past DBH (ib)} + 2 * (\text{bark thickness})$$

$$\text{and tree growth; i.e. dbh growth} = \text{current DBH(ob)} - \text{past DBH(ob)}$$

Real World Field Problem: to determine the dbh growth rate of trees in a forested area.

- Solution:
1. Measure the dbh, single bark thickness, 10 yr radial growth of sample trees.
 2. Compute current dbh (ib) using single bark thickness.
 3. Solve for the relationship of dbh (ob) and dbh (ib): $dbh_{ob} = b_0 + b_1 (dbh_{ib})$
 4. Compute past dbh(ib) = current dbh(ib) - 2*(radial growth)
 5. Compute past dbh(ob) as a function of past dbh (ib) with equation.
 6. Compute dbh growth as a function of past dbh(ob):
 7. Solve for the relationship between dbh growth and past dbh(ob):

Sample Tree Measurement:

1. dbh (ob) is measured with a d-tape in the field.
2. obtain single bark thickness with bark gauge.
3. obtain radial growth from core obtained with an increment borer.
4. measure total tree height with clinometer.

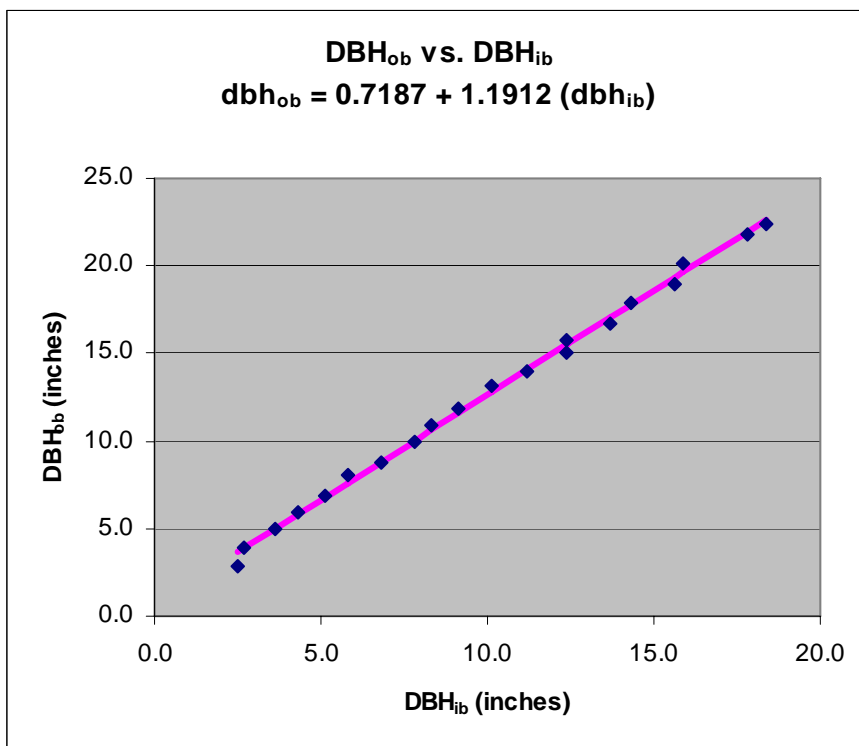
Tree#	dbh(ob)	Height	1x-Bark	1x-RG10	curr dib	Past dib	Past DBH	dbh gro
1	2.9	33	0.20	0.04	2.5	2.4	3.6	-0.70
2	3.9	40	0.60	0.11	2.7	2.5	3.7	0.23
3	5.0	45	0.70	0.15	3.6	3.3	4.6	0.35
4	5.9	49	0.80	0.20	4.3	3.9	5.4	0.54
5	6.9	59	0.90	0.23	5.1	4.6	6.2	0.65
6	8.0	58	1.10	0.23	5.8	5.3	7.1	0.92
7	8.8	65	1.00	0.30	6.8	6.2	8.1	0.70
8	10.0	69	1.10	0.33	7.8	7.1	9.2	0.78
9	10.9	64	1.30	0.33	8.3	7.6	9.8	1.08
10	11.9	73	1.40	0.40	9.1	8.3	10.6	1.29
11	13.1	70	1.50	0.46	10.1	9.2	11.7	1.45
12	14.0	77	1.40	0.45	11.2	10.3	13.0	1.01
13	15.0	78	1.30	0.42	12.4	11.6	14.5	0.51
14	15.8	73	1.70	0.47	12.4	11.5	14.4	1.43
15	16.7	80	1.50	0.54	13.7	12.6	15.8	0.95
16	17.9	80	1.80	0.43	14.3	13.4	16.7	1.17
17	19.0	83	1.70	0.60	15.6	14.4	17.9	1.13
18	20.1	72	2.10	0.67	15.9	14.6	18.1	2.04
19	21.8	90	2.00	0.66	17.8	16.5	20.4	1.45
20	22.4	85	2.00	0.56	18.4	17.3	21.3	1.10
	Y				X		X	Y

Sample Tree Calculations:

1. $dbh(ib) = dbh(ob) - 2*(1x-bark)$
2. fit regression of dbh (ob) as a linear function of dbh (ib) [see next worksheet]
 $dbh_{ob} = b_0 + b_1(dbh_{ib})$
3. $past\ dbh(ib) = current\ dbh(ib) - 2*(1x-radial\ growth)$
4. compute past dbh (ob) with the regression equation (step 2) from past dbh (ib)
5. calculate dbh growth = current dbh (ob) - past dbh (ob)
6. fit regression of dbh growth as a linear function of past dbh (ob) [see worksheet]
 $dbh\ growth = b_0 + b_1(past\ dbh_{ob})$
7. fit regression of $\ln(\text{height})$ as a linear function of $(dbh)^{-1}$:
 $Ln(H_0) = b_0 - b_1(dbh^{-1})$

Regression worksheet of: $dbh_{ob} = b_0 + b_1(dbh_{ib})$

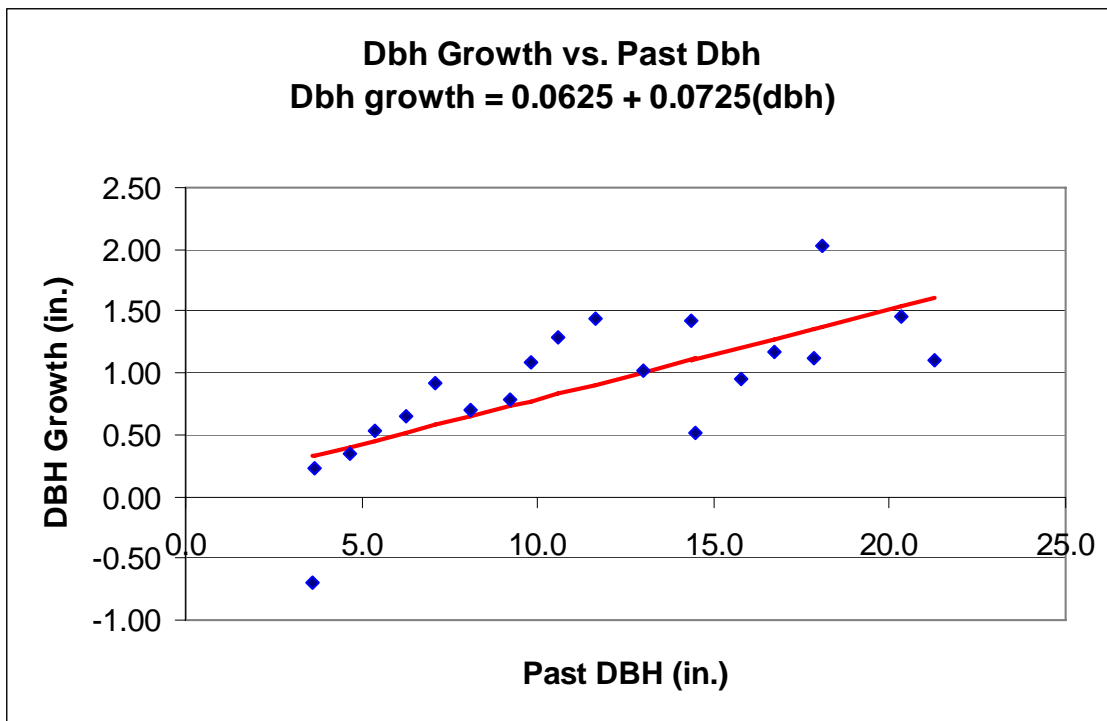
Tree#	dbh(ob)	curr dib	Y2	X2	XY	(y-y [^]) ²
1	2.9	2.5	8.41	6.25	7.25	0.63482
2	3.9	2.7	15.21	7.29	10.53	0.00123
3	5.0	3.6	25.00	12.96	18.00	0.00005
4	5.9	4.3	34.81	18.49	25.37	0.00348
5	6.9	5.1	47.61	26.01	35.19	0.01124
6	8.0	5.8	64.00	33.64	46.40	0.13850
7	8.8	6.8	77.44	46.24	59.84	0.00036
8	10.0	7.8	100.00	60.84	78.00	0.00011
9	10.9	8.3	118.81	68.89	90.47	0.08648
10	11.9	9.1	141.61	82.81	108.29	0.11633
11	13.1	10.1	171.61	102.01	132.31	0.12239
12	14.0	11.2	196.00	125.44	156.80	0.00366
13	15.0	12.4	225.00	153.76	186.00	0.24011
14	15.8	12.4	249.64	153.76	195.92	0.09610
15	16.7	13.7	278.89	187.69	228.79	0.11466
16	17.9	14.3	320.41	204.49	255.97	0.02150
17	19.0	15.6	361.00	243.36	296.40	0.09118
18	20.1	15.9	404.01	252.81	319.59	0.19418
19	21.8	17.8	475.24	316.84	388.04	0.01505
20	22.4	18.4	501.76	338.56	412.16	0.05637
Sum	250.0	197.8	3816.46	2442.14	3051.32	1.94781
mean	12.5	9.9				
CSS			691.46	485.90	578.82	
b₁	1.1912					
b₀	0.7187					
r²	0.9997					
s_{y,x}	0.3290					



Worksheet for: $dbh\ growth = b_0 + b_1(past\ dbh_{ob})$

Assumption: trees of the past dbh (ob) grew the amount of the dbh growth, not the current dbh trees (ob).

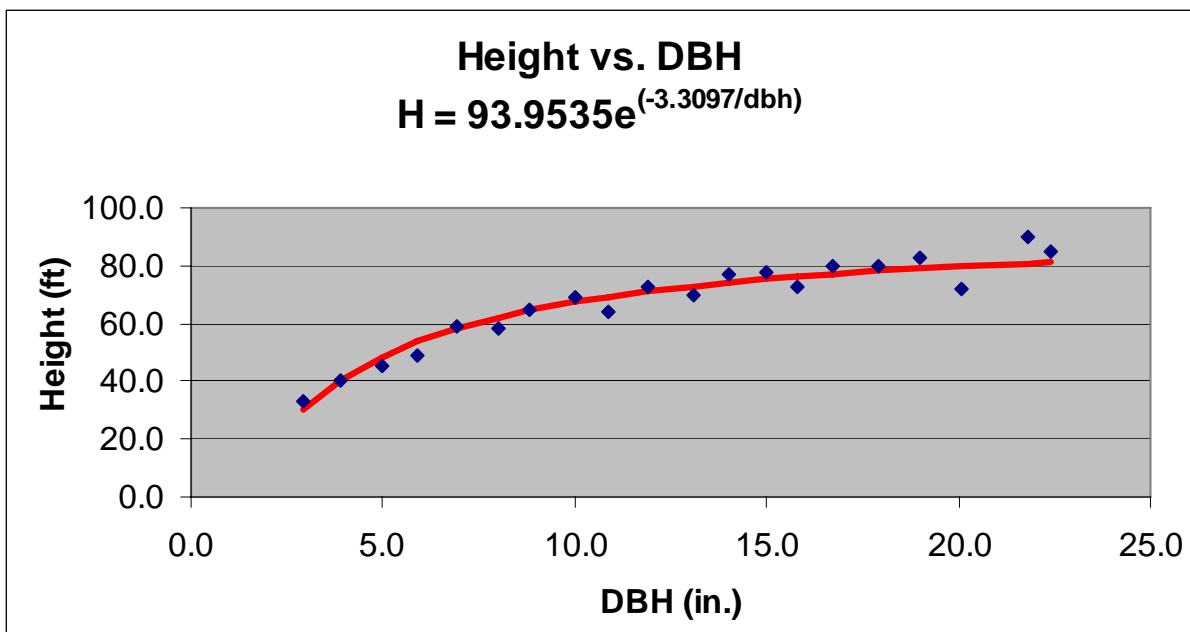
Y=dbh 10growth	X=past dbh(ob)		Y ²	X ₂	XY
-0.70	3.6		0.49	12.97	-2.5263
0.23	3.7		0.05	13.49	0.8340
0.35	4.6		0.12	21.62	1.6286
0.54	5.4		0.29	28.78	2.8728
0.65	6.2		0.43	39.01	4.0849
0.92	7.1		0.85	50.12	6.5144
0.70	8.1		0.48	65.68	5.6379
0.78	9.2		0.60	85.08	7.1570
1.08	9.8		1.17	96.43	10.6081
1.29	10.6		1.67	112.49	13.7248
1.45	11.7		2.09	135.82	16.8494
1.01	13.0		1.02	168.70	13.1390
0.51	14.5		0.26	209.94	7.3988
1.43	14.4		2.04	206.50	20.5460
0.95	15.8		0.90	248.13	14.9317
1.17	16.7		1.37	279.86	19.5913
1.13	17.9		1.27	319.43	20.1515
2.04	18.1		4.15	326.27	36.7931
1.45	20.4		2.10	414.13	29.5027
1.10	21.3		1.20	453.83	23.3644
18.1	231.9	Sums	22.57	3288.28	252.8040
0.9	11.6				
		CSS	6.261903	598.4558	43.37115
		b₁	0.0725		
		b₀	0.0625		
		r²	0.501954		
		s_{y,x}	1.765989		



Worksheet for:

$$\ln(H_0) = b_0 - b_1(\text{dbh}^{-1})$$

tree #	DBH	Ht	X=1/DBH	Y=ln(Ht)	X ²	Y ₂	XY	(Y - Y ₂) ²	Ŷ
1	2.9	33.0	0.3448	3.4965	0.11891	12.22557	1.20569	0.0090	30.01
2	3.9	40.0	0.2564	3.6889	0.06575	13.60783	0.94587	0.0000	40.21
3	5.0	45.0	0.2000	3.8067	0.04000	14.49068	0.76133	0.0055	48.47
4	5.9	49.0	0.1695	3.8918	0.02873	15.14627	0.65963	0.0081	53.62
5	6.9	59.0	0.1449	4.0775	0.02100	16.62631	0.59095	0.0002	58.16
6	8.0	58.0	0.1250	4.0604	0.01563	16.48720	0.50756	0.0047	62.12
7	8.8	65.0	0.1136	4.1744	0.01291	17.42551	0.47436	0.0001	64.51
8	10.0	69.0	0.1000	4.2341	0.01000	17.92766	0.42341	0.0005	67.48
9	10.9	64.0	0.0917	4.1589	0.00842	17.29631	0.38155	0.0065	69.35
10	11.9	73.0	0.0840	4.2905	0.00706	18.40804	0.36054	0.0007	71.15
11	13.1	70.0	0.0763	4.2485	0.00583	18.04971	0.32431	0.0017	72.98
12	14.0	77.0	0.0714	4.3438	0.00510	18.86865	0.31027	0.0014	74.18
13	15.0	78.0	0.0667	4.3567	0.00444	18.98091	0.29045	0.0012	75.35
14	15.8	73.0	0.0633	4.2905	0.00401	18.40804	0.27155	0.0018	76.20
15	16.7	80.0	0.0599	4.3820	0.00359	19.20216	0.26240	0.0014	77.07
16	17.9	80.0	0.0559	4.3820	0.00312	19.20216	0.24481	0.0006	78.10
17	19.0	83.0	0.0526	4.4188	0.00277	19.52615	0.23257	0.0025	78.94
18	20.1	72.0	0.0498	4.2767	0.00248	18.28987	0.21277	0.0103	79.69
19	21.8	90.0	0.0459	4.4998	0.00210	20.24829	0.20641	0.0118	80.72
20	22.4	85.0	0.0446	4.4427	0.00199	19.73715	0.19833	0.0023	81.05
20			Sum	2.2164	83.5212	0.36383	350.15446	8.86476	0.0703
n			Mean	0.1108	4.1761				
			CSS			0.11820	1.36511	-0.39121	
			b1			-3.3097			
			b0			4.5428			
			r2			0.9485			
			sy.x			0.0625	1.0644981		



Using the growth equation: $dbh\ growth = 0.0625 + 0.0725(past\ dbh_{ob})$

Using the height equation: $Ln(H_0) = 4.5428 - 3.3097\left(\frac{1}{dbh}\right)$

$$H_0 = 93.9635 e^{\left(\frac{-3.3097}{dbh}\right)}$$

Using a single tree volume equation: $Ft^3 (ob) = -.09 + .002618(D^2H_0)$

Result: A future stand and stock table that contains number of trees and volume by dbh class.

dbh	growth	Current Trees/acre	Future Trees/acre	Total Future Trees	Height	Volume / tree	Volume /acre
6	0.48	100	52	52	54.1	5.01	261
7	0.52	90	48+43	91	58.6	7.42	675
8	0.59	80	47+33	80	62.1	10.32	826
9	0.65	70	47+24	71	65.0	13.70	973
10	0.72	60	46+17	63	67.5	17.58	1,107
11	0.78	0	43	43	69.5	21.94	943
total		400	400	400			4,785

Stand Table growth Projection (STP): trees moving from a dbh class to another is a function of the dbh increment and the growth rate:

$$Growth\ ratio\ index_{dbh_i} = \frac{growth\ rate_{dbh_i}}{dbh\ increment} = n.f = n\ classes.percent$$

thus, f% of the trees move up n+1 class in the ith dbh class.

$$Growth\ ratio\ index_{dbh_i} = n+1\ classes . percent\ movement$$

Example: 6" class g.i.r = 0.48/1.0 = 0.48

The left side of the decimal in 0.48 means 0+1 = 1 dbh class

The right side of the decimal in 0.48 mean 48% move up 1 dbh class

Thus, 48% of 100 trees (i.e. 48 trees) move up 1 dbh class to 7 inches.

So, 52% of 100 trees (i.e. 52 trees) move up 0 dbh classes.