FO-2213 Forest Measurement
Topic 8: **Volume and Weight of Logs and Trees**

Chapter 5

Log: a portion of a tree that is 8 ft or more in length.

Sticks: a portion of a tree that is less than 8 ft in length.

Scaling: the process of measuring volumes of individual logs/sticks.

Units of Measure:

cubic foot = ft^3 = 12 in. x 12 in. x 12 in. = 1,728 cubic inches.

cubic meter = m^3 = 35.3 ft^3

cord = stack of wood that is 8 ft long x 4 ft high that contains 4 ft sticks
= 128 ft^3 of wood, bark, and airspace.

board foot = a plank that is 1 in. thick, 12 in. wide, and 1 ft long
= Thickness (inches) x Width (inches) x Length (ft)/12

Log Volumes:

Cross section Area (ft^2) = **B**= 0.005454 D^2, where D is diameter in inches.

Geometric solids
Paraboloid= ½ Area x length
Conoid = 1/3 Area x length
Neiloid = 1/4 Ares x length

Cubic volume of logs
Smalian's cubic volume: $ft^3 = [(B + b)/2] L$

Huber's cubic volume: $ft^3 = [(B_{12})] L$

Newton cubic volume: $ft^3 = [(B + 4 B_{1/2} + b)/6] L$

where: B = cross sectional area at the large end of the log.
$B_{1/2}$ = cross sectional area at the midpoint of the log.
b = cross sectional area at the small end of the log.

Stacked Wood: (per cord)

Gross cord ft^3= [Width (ft) x Height (ft) x Length (ft) /] 128] ft^3 of wood, bark, airspace

Wood only $ft^3 \doteq$(ave bolt diameter$_{ib}$ - 3)* 2 + 66   for loblolly pine   (+70 for longleaf)

Wood and bark $ft^3 \doteq$(ave bolt diameter$_{ib}$ - 3)* 2 + 80

Mid-Diameter	Wood only	Wood and Bark
5	70	84
6	72	86
7	74	88
8	76	90
9	78	92
10	80	94
11	82	96
12	84	98
13	86	100
14	88	102
15	90	104

Mississippi Weights and Measures Law: specifies that a cord of loblolly pulpwood will have a weight of 5,200 lbs (2.6 tons) and hardwood will be 5,600 lbs (2.8 tons) unless the buyer has research data to substantiate other values.

Weight Scaling of Wood:

Wood density and weight ratios:

$$Density\left(\frac{lbs}{ft^3}\right) = sp.gr. \, x \, 62.4\left(1 + \frac{\% moisture\ content}{100}\right)$$

Example: loblolly pine has a published specific gravity of 0.46 with a mc of 110%

Density = 0.46 x 62.4 (1 + 1.1)   = 60.2 lbs/ft^3 x 76 ft^3/cord =   4,575 lbs, wood only
+   700 lbs   barl
= total   5,275 lbs /cord

Rule of Thumb for green pine wood: 5,200 lbs / 85 ft^3/cord = 61.2 lbs/ft^3 wood and bark

Board Feet Log Rules:

Scaling diameter (D) = diameter inside bark at the small end of the log rounded to the nearest 1 inch class.

Scaling length (L) = length of the log in increments of 2 ft and the log must have a minimum of 0.3 ft of trim per log **.**

Equations:

Doyle Board Feet	$= (D - 4)^2L/16$	
Scribner Board Feet	$= (0.79D^2 - 2D - 4)L/16$	
International 1/4 Board Feet	$= 0.199D^2 - 0.6420D$	4 ft. log
	$= 0.398D^2 - 1.0850D - 0.2713$	8 ft. log
	$= 0.597D^2 - 1.3290D - 0.7143$	12 ft. log
	$= 0.796D^2 - 1.3740D - 1.2295$	16 ft. log

Int'l 1/4 Equation Taper = 0.5 inches per 4 ft. section

There are no direct equations for Int'l 1/4 inch volumes for log lengths that are not multiples of 4 ft., so you must use the 4 ft. equation or a portion of it and then increase the scaling diameter of the remaining log segment by 0.5 inches per 4 ft. and utilize the appropriate log equation above. The equations for International 1/4 log volume are thus:

10 ft. log $= 0.5(0.199D^2 - 0.6420D) + 0.398(D+0.25)^2 - 1.0850(D+0.25) - 0.2713$
14 ft. log $= 0.5(0.199D^2 - 0.6420D) + 0.597(D+0.25)^2 - 1.3290(D+0.25) - 0.7143$
18 ft. log $= 0.5(0.199D^2 - 0.6420D) + 0.796(D+0.25)^2 - 1.3740(D+0.25) - 1.2295$
20 ft. log $= [0.199D^2 - 0.6420D] + 0.796(D+0.5)^2 - 1.3740(D+0.5) - 1.2295$

Stand Tree Volume Equations:

Merrifield, R.G. and R.R. Foil. 1960. Volume equations for southern pine pulpwood. LSU Hill Farm Facts Bulletin Forestry 7, March 19678

Total Height: (FC =82)

Cords = -0.0361 + 0.00832 D -0.0000032 H + 0.0000235 D^2H

Cu. ft = -2.1210 + 0.60820 D -0.0242000 H + 0.0020300 D^2H

Merchantable Height (ft): (FC =82)

Cords = -0.0285 + 0.00712 D -0.0002700 H + 0.0000266 D^2H

Cu. ft = -2.0100 + 0.55130 D -0.0058510 H + 0.0023120 D^2H

G.S. Lee and R.C. Parker. 2003. Standing tree weight and volume tables for natural loblolly pine at the first delivery point. FWRC Bulletin FO-222, Mississippi State University, 14 pp.

Regression equation coefficients and fit statistics for predicting weight and sample profile volumes of loblolly pine on the John W. Starr Memorial Forest, Oktibbeha County, Mississippi.

Coefficients	Weight and Volume Equations*						
	Pounds	Ft^3 ob	Ft^3 ib	Cords	Doyle BF	Scribner BF	Int-1/4" BF
β_0	0.284043	0.004315	0.003207	0.000117	0.001102	0.004858	0.007174
β_1	1.993407	2.012104	2.028062	1.651819	2.684712	2.282177	2.170757
β_2	0.866926	0.896237	0.923953	0.928341	1.094743	1.074633	1.084732
β_3	0.500000	0.498399	0.508773	0.465495	1.002157	0.881074	0.862677
β_4	1.140000	1.149355	1.136078	1.063535	2.052055	1.897239	1.890456
n	103	568	568	568	394	394	394
$S_{y.x}$	451.677	0.705	0.579	0.010	12.162	8.550	6.721
I^2	97.46%	99.99%	99.99%	99.97%	99.89%	99.95%	99.97%

* $Y = b_0\ DBH^{b_1}\ MH^{b_2}\ e^{b_3\left(\frac{MTD}{DBH}\right)^{b_4}}$ where: Y = tree weight or volume, MH = merchantable height in feet, MTD = merchantable top diameter in inches, and b_I = regression parameters for the respective weight/volume equation.

Pine Volume Equations:

DOYLE to 8" Top = -42.46 + .009747 D^2H_0

Ft^3 (ob) to 8" Top = -7.19 + .002639 D^2H_0

Ft^3 (ob) to 3" Top = -.09 + .002618 D^2H_0

DOYLE to 10" Top = -118.89 + .019422 D^2H_4

Ft^3 (ob) to 10" Top = -17.10 + .003957 D^2H_4

Ft^3 (ob) to 4" Top = 1.41 + .003948 D^2H_4

where H_0 = total height in ft and H_4 = height (ft) to 4 inch top

Standing Tree Weights

Average conversion factors do not properly assess weight of standing trees. It is best to use a single-tree, weight function based on the tree's dimensions of dbh and height.

Examples of single-tree weight functions:

S.W. MS and N. LA: $tons/tree = 0.001099\, D^{2.318201}\; MH^{0.588348}$

where MH = No. of 16 ft logs and half logs (1.0, 1.5, etc.)

John Starr Forest:

$$tons/tree = 0.000142022[dbh^{1.993407}\; MH^{0.866926}]\exp[0.500(\frac{MTD}{dbh})^{1.14}]$$

where MH = merchantable height in feet to specified top
MTD = merchantable top diameter in inches

Developing Standing Tree Volumes:

Standing tree volume equations/tables are developed from felled tree data. The tree is felled and measured in 4-8 ft sections. The volume of each section is computed in the desired unit of measure and then summed for the total tree.

Felled Tree Tally Sheet

Tree 1 dbh 11.8 Total Height 60 Height 3" 54.3 Height 6" 41.7

Bolt No.	Cum. ht.	Dia-ob	Single Bark	Dia-ib	Bolt VOB	Bolt VIB	Scaling Dia	Doyle Volume
St.	0.5	12.9	0.8	11.3	0	0		
1	4.5	11.8	0.6	10.7	3.33	2.64		
2	8.5	11.3	0.6	10.1	2.91	2.36	10.7	22.44
3	12.5	10.7	0.6	9.6	2.64	2.12		
4	16.5	10.1	0.5	9.2	2.36	1.93	9.65	15.96
5	20.5	9.6	0.4	8.8	2.12	1.77		
6	24.5	9.0	0.4	8.3	1.89	1.60	8.65	10.81
7	28.5	8.4	0.4	7.6	1.65	1.38		
8	32.5	7.7	0.3	7.1	1.42	1.18	7.4	5.78
9	36.5	7.0	0.4	6.3	1.18	0.98		
10	40.5	6.3	0.4	5.5	0.97	0.76	5.9	1.81
11	44.5	5.3	0.3	4.8	0.74	0.58		
12	48.5	4.2	0.2	3.8	0.50	0.41		
13	52.5	4.0	0.8	2.5	0.37	0.23		
CALC	54.3	3.0	0.2	2.6	0.12	0.06		
14	56.5	1.8	0.2	1.4				
TOTAL					CV3" ob 22.20	CV3" ib 18.00	CV6" ob 20.72	Doyle 56.5

Height to 3 inch top, ob. = Height to top of Bolt 13 + (1.0/2.2)* Length of Bolt 14
= 52.5 +1.8 = 54.3

Or (4.0 - 1.8)/4 = (4.0- 3.0)/ x , thus x= 1.9 ft.

Felled Tree Data:

Tree#	DBH	Height	Hgt to 6"	CV3, ob	CV3, ib	CV6, ob	Doyle
1	7.80	64.90	31.17	11.85	9.85	8.97	13.11
2	8.30	77.50	68.40	16.04	13.45	16.26	25.21
3	9.30	71.50	44.50	17.25	14.57	14.58	29.25
4	9.50	77.40	74.80	37.59	20.36	23.19	49.80
5	10.60	84.20	60.60	27.20	22.94	26.56	61.56
6	10.80	75.00	48.50	17.86	15.35	19.39	31.09
7	11.10	86.50	68.50	30.23	24.45	28.76	129.74
8	11.20	87.00	67.83	33.53	26.30	32.28	90.49
9	11.30	85.05	65.33	35.23	29.11	34.05	51.47
10	11.30	79.70	68.40	31.93	26.75	31.40	87.07
11	12.30	90.60	74.80	37.59	31.92	36.37	110.44
12	12.50	95.30	69.30	38.91	32.70	37.82	128.10
13	12.50	86.45	66.86	37.83	30.90	36.65	57.41
14	12.90	87.00	67.38	41.03	35.73	39.80	69.25
15	13.00	92.10	76.45	47.44	38.83	46.45	178.19
16	13.20	96.00	72.83	47.72	38.66	46.69	157.86
17	14.60	90.47	75.67	57.63	48.97	56.73	107.03
18	14.90	97.20	82.13	67.42	57.36	66.64	140.97
19	15.00	86.00	66.80	34.15	28.75	32.77	94.96
20	16.20	96.00	79.67	73.70	62.29	36.29	154.43

Solve for regression coefficients for model: $Vol_{di} = b_0 + b_1(D^2 H)$
where di = top diameter (ib or ob)

Example: $ft^3 Vol_6 = 7.798 + 0.002002(D^2 H_0)$ with R^2 =0.65 and $s_{y.x}$= ± 8.42 ft^3

Profile Equations: Allow the computation of diameters at any point on the main stem, then the volume of the tree segment can be computed with any desired volume unit/function. Parker, R.C. and T.G. Matney. 1999. Comparison of Optical Dendrometers for Prediction of Standing Tree Volume. Southern Jour. Appl. Forestry 23(2):100-107

At stump height of 0.5 feet (i.e., h_i = 0.5 ft)

$$\frac{dob_{0.5}}{dbh_{ob}} = 0.9364\left(\frac{1}{dbh_{ob}}\right) + 1.1294$$

with n=96, $s_{y.x}$=0.0708 and I^2 = 0.1165

Below breast height (i.e., h_i < 4.5 ft)

$$y_i = 1 + -7.1727(x_i - w_i) + 45.3420(x_i^2 - w_i^2)$$

with n=192, $s_{y.x}$=0.0567 and I^2 = 0.6691

where: $y_i = \frac{dob_{h_i}}{dbh_{ob}}$ $x_i = \frac{h_i}{h_t}$ $w_i = \frac{4.5}{h_t}$

Bark relationship above breast height (i.e., h_i > 4.5 ft)

$$\left(\frac{dib_{h_i}}{dob_{h_i}}\right) = \left(\frac{dbh_{ib}}{dbh_{ob}}\right) e^{0.0642\left[\frac{dbh_{ob}}{dob_{h_i}} - 1\right]^{0.25}}$$

with n=1,600, $s_{y.x}$=0.0368 and I^2 = -0.5791

Above breast height (i.e., h_i > 4.5 ft)

$$y_i = \frac{x_i - 1}{w_i - 1} + 1.74963\left[(x_i^2 - 1) - \frac{(w_i^2 - 1)(x_i - 1)}{(w_i - 1)}\right] - 1.5837\left[(x_i^3 - 1) - \frac{(w_i^3 - 1)(x_i - 1)}{(w_i - 1)}\right]$$

where: $y_i = \frac{dob_{h_i}}{dbh_{ob}}$ $x_i = \frac{h_i}{h_t}$ $w_i = \frac{4.5}{h_t}$

with n= 1,699 $s_{y.x}$ = 0.0457 and I^2 = 0.9665

Sample tree computations with profile equations:

dbh	**14.1**									
height	**96**									
h_i	**x_i**	**w_i**	**dob_i**	**1x Bark**	**dib**	**FC%**	**Cu. ft 3"**	**Cu. Ft 6"**	**Doyle BF**	**bf:ft^3**
0.5			16.9	1.1	14.6					
4.5			14.1	1.0	12.2		5.3	5.3		
8.5	0.088542	0.046875	13.5	0.9	11.7		4.2	4.2	37.0	
16.5	0.171875	0.046875	12.5	0.8	10.8	76.7%	7.4	7.4	29.6	
24.5	0.255208	0.046875	11.7	0.8	10.1		6.4	6.4	23.8	
32.5	0.338542	0.046875	11.0	0.7	9.5		5.6	5.6	19.8	
40.5	0.421875	0.046875	10.3	0.7	8.9		5.0	5.0	15.7	
48.5	0.505208	0.046875	9.6	0.6	8.3		4.3	4.3	12.5	
56.5	0.588542	0.046875	8.7	0.6	7.6		3.7	3.7	8.4	
64.5	0.671875	0.046875	7.7	0.5	6.7		2.9	2.9	5.1	
74.0	0.770833	0.046875	6.0	0.4	5.3		2.5	2.5	1.5	
80.5	0.838542	0.046875	4.6	0.3	4.1		1.0			
86.7	0.903125	0.046875	3.0	0.1	2.7		0.5			
						Sum	48.7	47.2	153.5	3.25