

**FO-2213 Forest Measurement
Topic 9: Regressions of Tree Volume on D²H**

Chapter 8

Combined Variable Model:

The model described by Spurr (1956) is the most common standing tree volume model used.

$$Volume = b_0 + b_1 (D^2 H^1) \quad \text{where} \quad \text{volume} = \text{any volume/weight unit}$$

D = dbh in inches

H = height to defined top diameter

The coefficient of H is assumed to be 1.

Examples:

$$\begin{aligned} \text{Ft}^3 \text{ (ob) to 3" Top} &= -.09 + .002618 D^2 H_0 \\ \text{Ft}^3 \text{ (ob) to 8" Top} &= -7.19 + .002639 D^2 H_0 \\ \text{DOYLE to 8" Top} &= -42.46 + .009747 D^2 H_0 \end{aligned}$$

$$\begin{aligned} \text{Ft}^3 \text{ (ob) to 4" Top} &= 1.41 + .003948 D^2 H_4 \\ \text{Ft}^3 \text{ (ob) to 10" Top} &= -17.10 + .003957 D^2 H_4 \\ \text{DOYLE to 10" Top} &= -118.89 + .019422 D^2 H_4 \end{aligned}$$

$$\text{tons / tree} = 0.001099 D^{2.318201} MH^{0.588348}$$

where MH = No. of 16 ft logs and half logs (1.0, 1.5, etc.)

Non-Linear Combined Variable Model:

With the advent of non-linear regression packages, the coefficients of D and H can be solved for and not assumed to be 2 and 1, respectively.

$$Volume = b_0 + b_1 (D^{b_3} H^{b_4})$$

Examples: (from felled tree data in Topic 11)

$$\begin{aligned} CV_3 &= 3.60584 + 0.0026014(D^2 H); & I^2 &= 0.83, s_{y,x} = \pm 6.94 \text{ ft}^3 \\ CV_3 &= 14.2308 + 0.000000002179 (D^{2.13} H^{3.96}); & I^2 &= 0.87, s_{y,x} = \pm 6.39 \text{ ft}^3 \end{aligned}$$

$$\begin{aligned} CV_6 &= 7.7980 + 0.002002 (D^2 H_0); & I^2 &= 0.65, s_{y,x} = \pm 8.42 \text{ t}^3 \\ CV_6 &= 9.4442 + 0.002373 (D^2 H_6); & I^2 &= 0.68, s_{y,x} = \pm 8.08 \text{ ft}^3 \\ CV_6 &= -4.7610 + 0.040855 (D^{0.89} H_6^{1.10}); & I^2 &= 0.75, s_{y,x} = \pm 7.62 \text{ ft}^3 \end{aligned}$$

$$\text{Doyle} = 8.2392 + 0.007879 (D^2 H_6); \quad I^2 = 0.61, s_{y,x} = \pm 31.31 \text{ bd. ft}$$

- a. Thus, the gain in fit precision from (D²H) to (D^{b₂} H^{b₃}) is negligible.
- b. And, as you increase the merchantability limits (i.e. from 3" to 6" cubic, then Doyle), the fit statistics get poorer and poorer.
- c. The relationship between volume and height variables also affects precision; CV₆ -H₆

Wood Utilization Standards - Merchantable Top Diameters

Merchantability standards such as top diameters influence standing tree volume; few volume functions allow for variable top diameters. Either the height variable is defined for a fixed top diameter or the volume variable is defined for fixed top diameter.

The conventional method for handling top utilization diameters uses a total volume function that is then modified by a ratio function that utilizes a specified top diameter:

$$ft^3 = 0.34864 + 0.00232 (D^2H) \text{ for plantation loblolly pine (Burkhart, 1977)}$$

$$R = 1 - 0.32354 (d_t^{3.1579}/dbh^{2.7115}); \text{ note that when } d_t = 0 \text{ then } R=1.$$

Example: for a 10" dbh, 75 ft tree, total cubic volume is computed as:

$$Vol = 0.34864 + 0.00232 (10^2 75) = 17.75 \text{ ft}^3$$

The ratio of ft^3 to a 4 inch top diameter is computed as:

$$R = 1 - 0.32354 (4^{3.1579}/10^{2.7115}) = 0.95$$

Thus, the ft^3 to a 4 inch top diameter is computed as:

$$Vol_4 = (17.75) (0.95) = 16.86 \text{ ft}^3$$

The method developed by Lee and Parker (2003), incorporates the merchantable-to-total ratio into the volume function as:

$$Y = b_0 DBH^{b_1} MH^{b_2} e^{b_3 \left(\frac{MTD}{DBH}\right)^{b_4}}$$

which is a single step process that lends itself to computer operations. Thus for cubic volume to a 4-inch top, ob, and an estimated 60 ft to a 4-inch top; the equation would be:

$$ft_4^3 = 0.004315 (10^{2.012104}) (60^{0.896237}) e^{0.498399 \left(\frac{4}{10}\right)^{1.149355}} = 20.71 \text{ ft}^3$$

Weight of Standing Trees

Average Conversion Factor

The practice of applying average values of lbs/ft³ is not the best approach because the weight of standing trees varies with dbh and height.

If an average conversion factor is used, the conversion should represent the average tree size (i.e. dbh and height) in the data set. This means that an inventory should be processed to obtain average tree size then reprocessed with the average conversion factor for the average tree size.

Examples:

Pulpwood = 5,200 lbs/cord thus the average weight would be $5,200/75 \text{ ft}^3$ of wood and bark or approximately 69 lbs/ft³.

Chip'n saw = about the same weight as pulpwood, but usually a little higher because the sticks are straighter and more uniform in size. Thus more solid wood per cord, so $5,200/85 \text{ ft}^3 = 61 \text{ lbs/ft}^3$ which is a lower weight per ft³ but will yield a higher cord weight.

Sawtimber = generally computes as tons/mbf factor. For example if the tree size averaged 10 tons/mbf (mbf= ,000 board feet) and the average log contained 3.7 bd ft/ft³ then 1 mbf would have approximately 270 ft³. Thus, 1 ft^3 of sawtimber would weight $(10 * 2,000)/270 = 74 \text{ lbs per ft}^3$. If the tree size was 8 tons/mbf then the weight would be 59.2 lbs/ft³.

Sawtimber weights should be applied from a standing tree weight table/equation that assigns an appropriate weight to each dbh-height class.

Per Tree Weight Table

A standing tree weight table will apply the appropriate weight to various tree dbh-height classes.

tons - 3" top dbh	1	1.5	2	2.5	3	3.5	4	4.5	5
	16	24	32	40	48	56	64	72	80
6	0.0701	0.0997	0.1279	0.1552	0.1818	0.2078	0.2333	0.2584	0.2831
7	0.0919	0.1307	0.1677	0.2035	0.2383	0.2724	0.3058	0.3387	0.3711
8	0.1168	0.1660	0.2130	0.2585	0.3027	0.3460	0.3885	0.4303	0.4714
9	0.1447	0.2057	0.2639	0.3202	0.3751	0.4287	0.4813	0.5331	0.5840
10	0.1757	0.2497	0.3204	0.3887	0.4553	0.5204	0.5843	0.6471	0.7090
11	0.2097	0.2980	0.3824	0.4640	0.5434	0.6211	0.6974	0.7724	0.8462
12	0.2467	0.3506	0.4500	0.5460	0.6395	0.7309	0.8206	0.9088	0.9957
13	0.2868	0.4076	0.5231	0.6347	0.7434	0.8497	0.9540	1.0565	1.1576
14	0.3299	0.4689	0.6017	0.7302	0.8552	0.9775	1.0974	1.2154	1.3317
15	0.3761	0.5346	0.6860	0.8324	0.9749	1.1143	1.2511	1.3855	1.5181
16	0.4254	0.6045	0.7757	0.9413	1.1025	1.2601	1.4148	1.5669	1.7167
17	0.4776	0.6788	0.8711	1.0570	1.2380	1.4150	1.5886	1.7594	1.9277
18	0.5329	0.7574	0.9719	1.1794	1.3813	1.5788	1.7726	1.9632	2.1509
19	0.5913	0.8403	1.0784	1.3085	1.5326	1.7517	1.9667	2.1781	2.3864
20	0.6527	0.9276	1.1903	1.4444	1.6917	1.9336	2.1709	2.4043	2.6342
tons 6" top dbh	16	24	32	40	48	56	64	72	80
8	0.1422	0.2021	0.2593	0.3147	0.3686	0.4213	0.4730	0.5238	0.5739
9	0.1719	0.2443	0.3135	0.3804	0.4455	0.5092	0.5717	0.6331	0.6937
10	0.2046	0.2908	0.3732	0.4528	0.5304	0.6062	0.6806	0.7537	0.8258
11	0.2404	0.3417	0.4385	0.5320	0.6232	0.7122	0.7997	0.8856	0.9703
12	0.2793	0.3969	0.5093	0.6180	0.7238	0.8273	0.9289	1.0287	1.1271
13	0.3212	0.4564	0.5857	0.7107	0.8324	0.9514	1.0682	1.1831	1.2962
14	0.3661	0.5203	0.6677	0.8102	0.9489	1.0846	1.2177	1.3486	1.4776
15	0.4141	0.5885	0.7552	0.9163	1.0732	1.2267	1.3772	1.5253	1.6712
16	0.4651	0.6610	0.8482	1.0292	1.2055	1.3778	1.5469	1.7132	1.8770
17	0.5191	0.7378	0.9468	1.1488	1.3455	1.5379	1.7267	1.9123	2.0952
18	0.5762	0.8189	1.0508	1.2751	1.4935	1.7070	1.9165	2.1225	2.3255
19	0.6363	0.9043	1.1605	1.4082	1.6493	1.8851	2.1165	2.3440	2.5682
20	0.6995	0.9941	1.2756	1.5479	1.8130	2.0722	2.3265	2.5766	2.8230