

FO 4311/6311 - Spatial Technologies Laboratory 2  
**2A. Determination of Photo Scale: Point and Average**

**Materials Required:**

By Student: Engineer's Scale  
30°-60°-90° Triangle  
Calculator & pencil

By Lab: Contact prints AVE 2KK 91  
28159 178 174 & 180  
NAPP 3986 209 or 210  
USFWS 16 196 58 or 59  
Betheden 7.5 minute quad sheet  
Bradley 7.5 minute quad sheet

**Laboratory Objectives:**

The objectives of this laboratory exercise are to give the student:

1. a knowledge of scale definition and causes of scale variation on aerial images,
2. examples of "large" and "small" scales on imagery and maps,
3. a knowledge of and ability to calculate point and average image scale,
4. a knowledge of and ability to calculate ground distance from image scale, and
5. the ability to calculate image scale when focal length and/or flying height are known,

**Background Information:**

Photogrammetry is the science, art, and technology of obtaining reliable information about physical objects and the environment through processes of recording, measuring, and interpreting photographic images and patterns of recorded radiant electromagnetic energy, and other phenomena. Included in the definition of photogrammetry are two distinct areas:

1. Metric photogrammetry - making precise measurements from photos and other image media to determine relative locations of points, distances, scales, angles, areas, volumes, elevations, and the sizes and shapes of objects.
2. Interpretative photogrammetry - recognizing and identifying objects on aerial imagery and judging their significance.

Because of the geometry and various types of image distortion, an aerial photograph cannot be considered a map because:

### Map

1. map is a two-dimensional scale representation of a portion of the earth's surface.
2. all points appear as if they are viewed from above, straight down; i.e. orthogonal projection.
3. the scale is constant across the entire map and all points have been located, orthogonally, on the datum plane.

### Aerial Photo

1. aerial photo is a perspective or central projection.
2. a photo is a map for all points that are on the datum plane.
3. distortion results for all points not on the datum plane.
4. the scale is not constant across the entire photo.

An aerial image that has been exposed in a precision "mapping" camera can be treated analytically because the camera has been calibrated to certain precise metrical characteristics - primarily the format and the focal length. A camera is an **angle-recording instrument** based on the geometry of a single photo relative to the fiducial marks, principal point, conjugate principal point, nadir point, and isocenter.

Fiducial Marks - an indicator of the center of the film plane in the aerial camera. The marks appear in the middle of each photograph border.

Principal Point (PP) - the optical (and geometric) center of the film plane of the photograph. The principal point of each photograph is located by connecting the fiducial marks, at the center of the photograph. There is only one PP per photo.

Conjugate Principal Point (CPP) - the PP of an adjoining photograph for photos with at least 55% end overlap. Each photo of a stereo pair will have at least 2 CPP's. If sidelap is sufficient, there will be 3 CPP's along each side of the photo.

Nadir Point (NP) - a plumb point perpendicular to the 0 datum passing through the lens at the instant of exposure.

## Scale

Scale defines the relationship between a linear distance on a vertical photo and the corresponding actual distance on the ground.

**Scale = the ratio of a distance on an image or map to the corresponding distance on the ground.**

Representative Fraction is the most commonly used expression of scale and it will be used extensively in photogrammetry.

$$RF = \frac{1}{S} = \frac{d}{D}$$

where *RF* = representative fraction

*S* = denominator of *RF*

*d* = image or photo distance

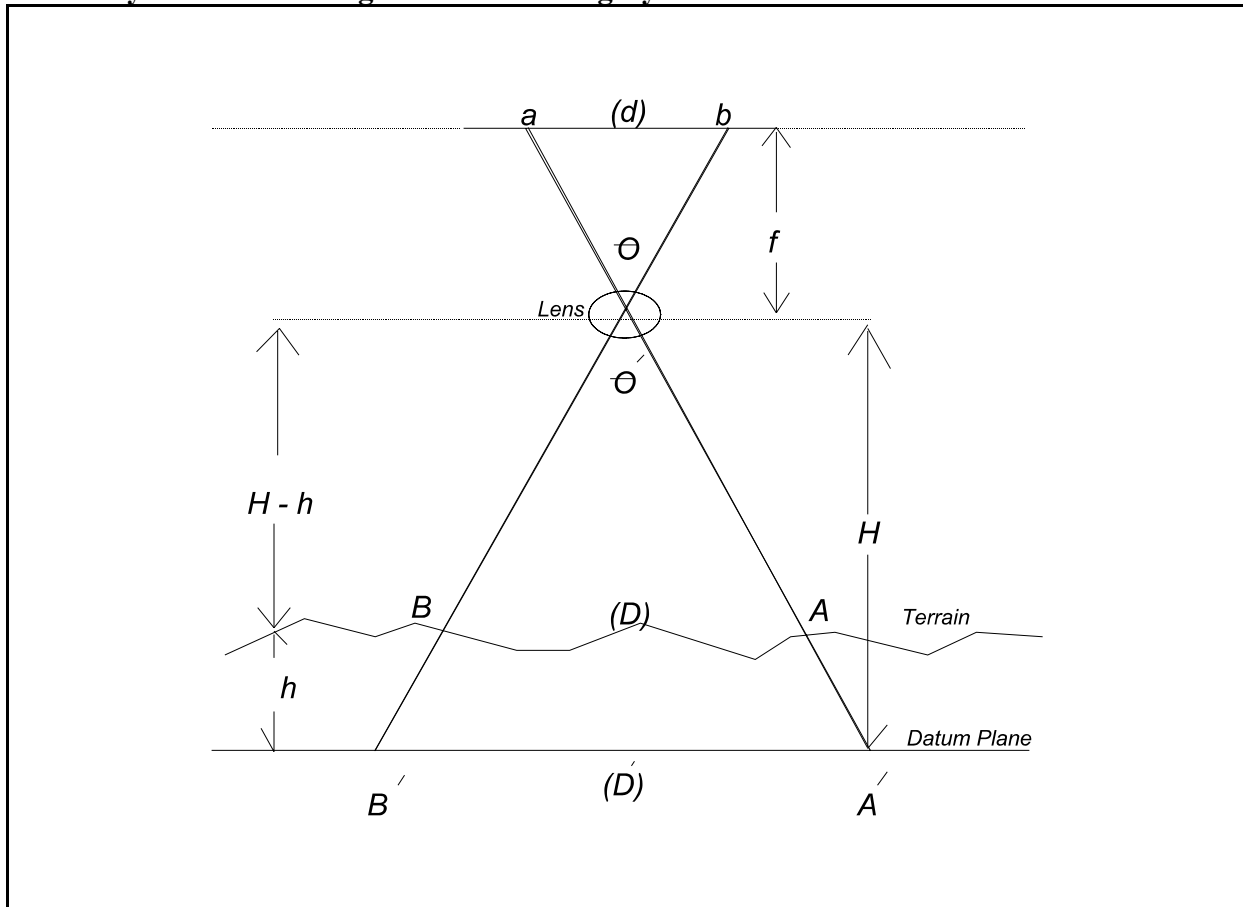
*D* = ground distance

The characteristics of an R.F. are

1. the **numerator** is always **1**; 1 over something.
2. it is **unitless**, until **you** assign units.
3. it indicates **1 unit** of horizontal distance on the image or map is equal to **X** number of the same units on the ground.

That is, 1/24,000 means that **1 photo unit equals 24,000 ground units**. You can look at it as **1 photo inch = 24,000 ground inches**. OR, 1 photo mm = 24,000 ground mm.; or 1 of anything to 24,000 of the same thing. ft/ft., m./m., chains/chains.

## Geometry of similar triangles on aerial imagery



The geometry of similar triangles in the figure above is used to derive the following relationships:

$$\Delta(abL) \approx \Delta(ABL) \quad \text{WHY?}$$

$$\text{since } \overline{ab} : \overline{AB}$$

$$d = \overline{ab} \quad (\text{the image distance})$$

$$D = \overline{AB} \quad (\text{the ground distance})$$

also  $f:H$

$$\text{thus: } \frac{d}{D} = \frac{f}{H-h} \quad \text{or} \quad \frac{d}{D'} = \frac{f}{H}$$

Since scale is the relationship between image distance ( $d$ ) and ground distance ( $D$ ), the scale relationship of an aerial photograph can be expressed as:

$$RF = \frac{1}{S} = \frac{d}{D} = \frac{f}{(H-h_0)}$$

where  $f$  = focal length

$H$  = platform height (m.s.l.)

$h_0$  = elevation of object above m.s.l.

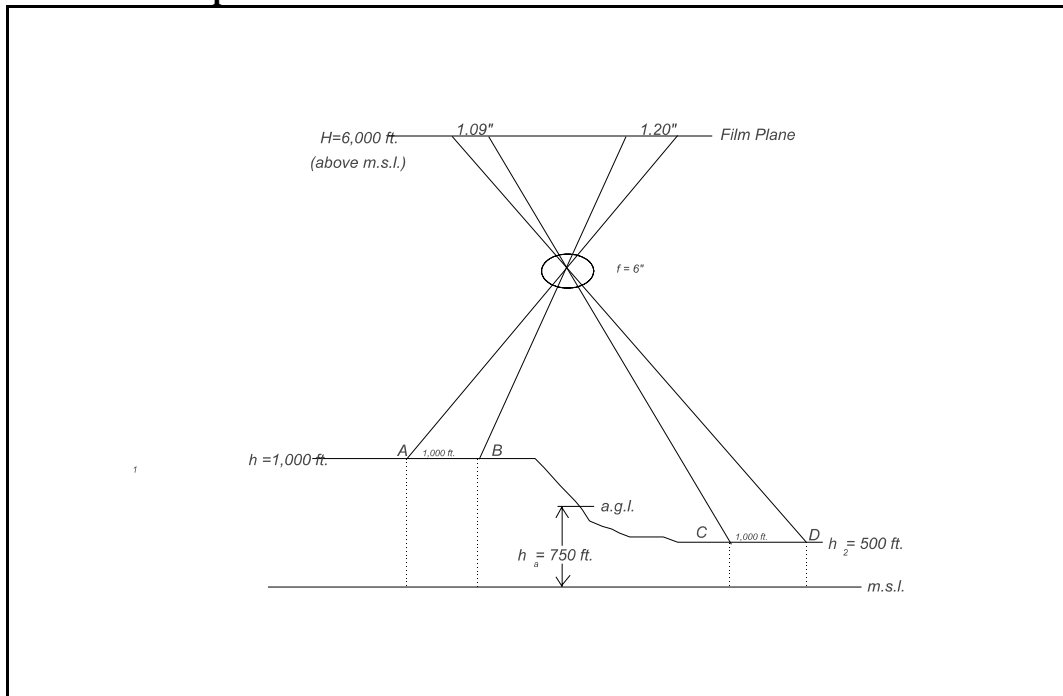
$H-h_0$  = flying height above object elevation

### Sources of Scale Variation:

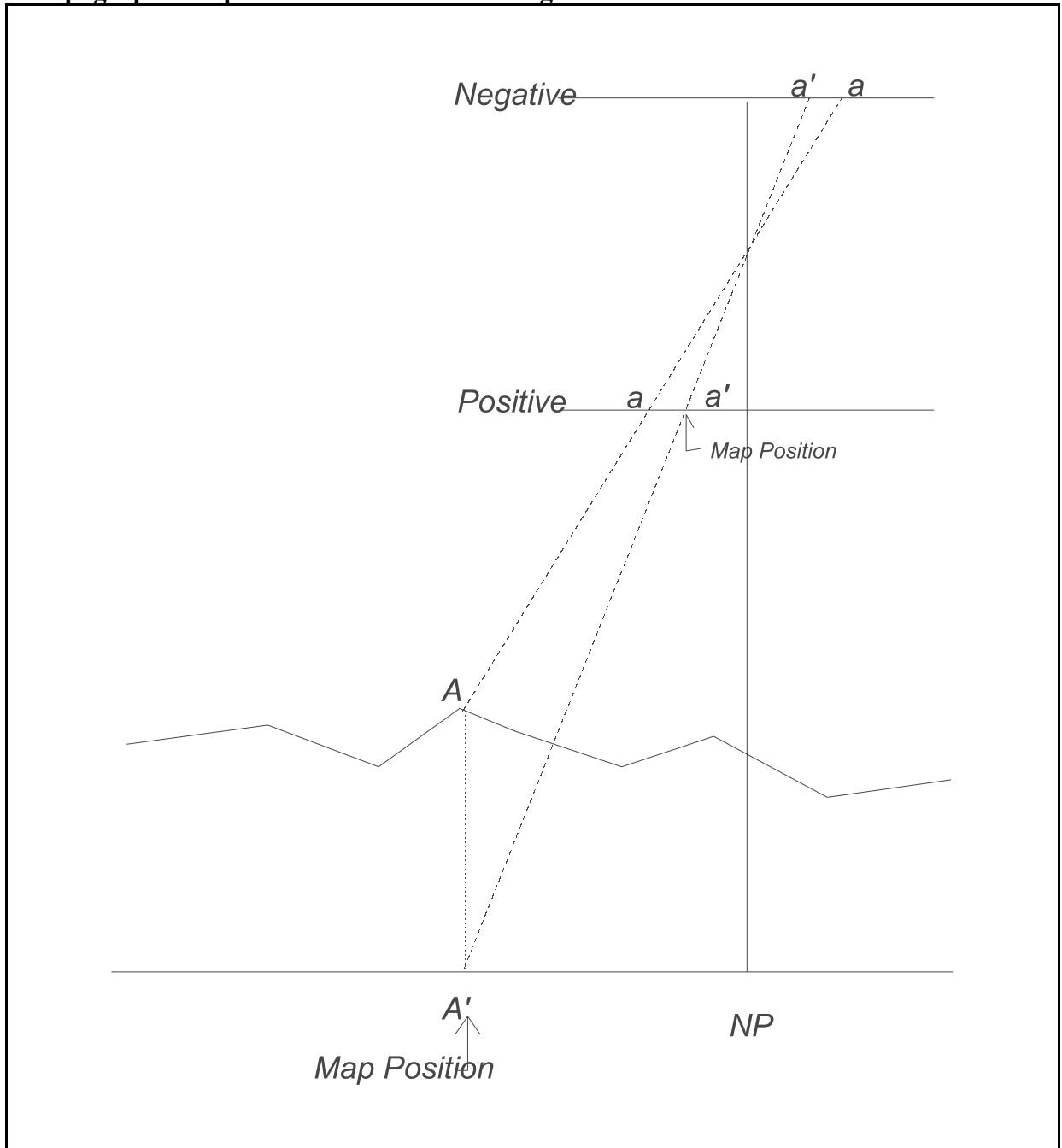
1. Terrain
2. Topographic Displacement
3. Focal length and flying Height
4. Tip and Tilt

Only if a **truly vertical image** is acquired over **perfectly flat terrain** is the scale of an aerial image uniform across the image!!

### **1. Variation in photo scale from terrain**



## 2. Topographic displacement on an aerial image.



## Scale Determination:

Scale variation due to terrain and topographic displacement is radial from the principal point thus in average scale determination, you should use/measure long diagonal lines that cross the radials of displacement.

### A. Average Scale

1) using  $\sum d_i$  and  $\sum D_i$

2) using  $f$  and  $H$  at  $h_{\text{average}}$

### B. Point scale

1) using  $d$  and  $D$

2) using  $f$  and  $H$  at  $h_0$

A. Average Scale: The average scale is obtained by averaging the measured photo and ground distances or averaging the terrain elevations imaged on a frame: /

1) Using the sum of photo and ground distances: (gives a weighted average not the scale at the average elevation!)

Thus the average scale is:  $1/S = d/D$

$$1/S = [(1.09+1.2)/12] / (1,000+1,000) = 1/10,480 \text{ (weighted average)}$$

2) Using average elevation: Average elevation =  $(1,000 + 500)/2 = 750$  ft.

Thus the average scale is:  $1/S = .5/(6,000-750) = 1/10,500$ .

### B. Point Scale:

1) Using  $d$  and  $D$ : (i.e. assuming both ends of line are at the same elevation)

$$\text{Scale AB} = 1/S = 1.2/1000 = 1/10,000$$

$$\text{Scale CD} = 1/S = 1.0909/1000 = 1/11,000$$

2) Using  $f$  and  $H$  at a known  $h$ : (i.e. a known/specific elevation)

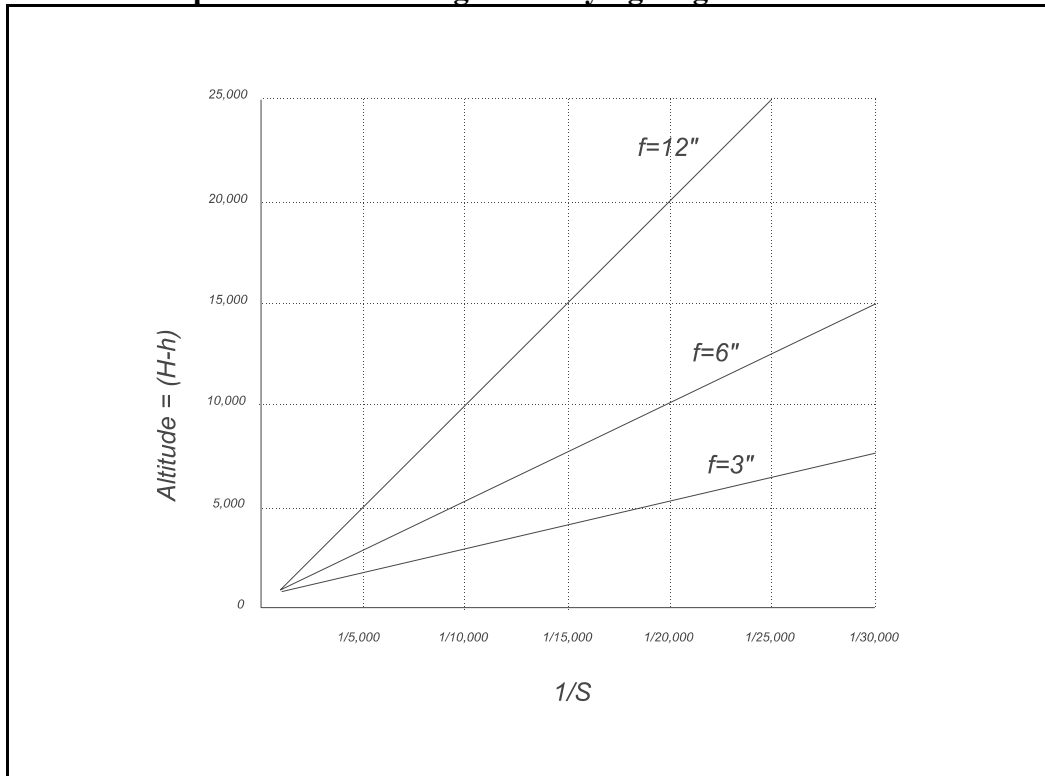
$$\text{Scale AB: } 1/S = 0.5/(6000 - 1000) = 1/10,000 \text{ or } 833 \text{ ft/in (using } f \text{ and } H-h)$$

$$1/S = 1.20"/12/1000 \text{ ft} = 1/10,000 \text{ or } 833 \text{ ft/in (using } d \text{ and } D)$$

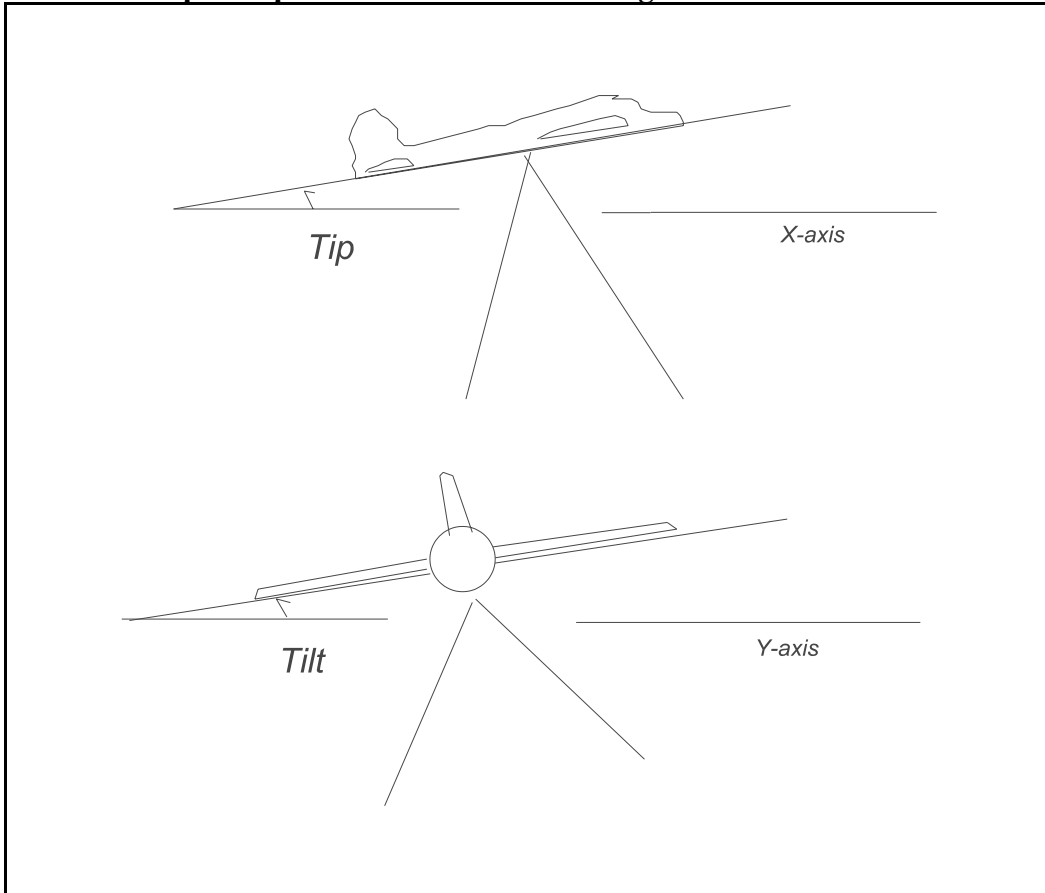
$$\text{Scale CD } 1/S = 0.5/(6000 - 500) = 1/11,000 \text{ or } 916.67 \text{ ft/in. (using } f \text{ and } H-h)$$

$$1/S = 1.09"/12/1000 \text{ ft} = 1/11,000 \text{ or } 916.67 \text{ ft/in (using } d \text{ and } D)$$

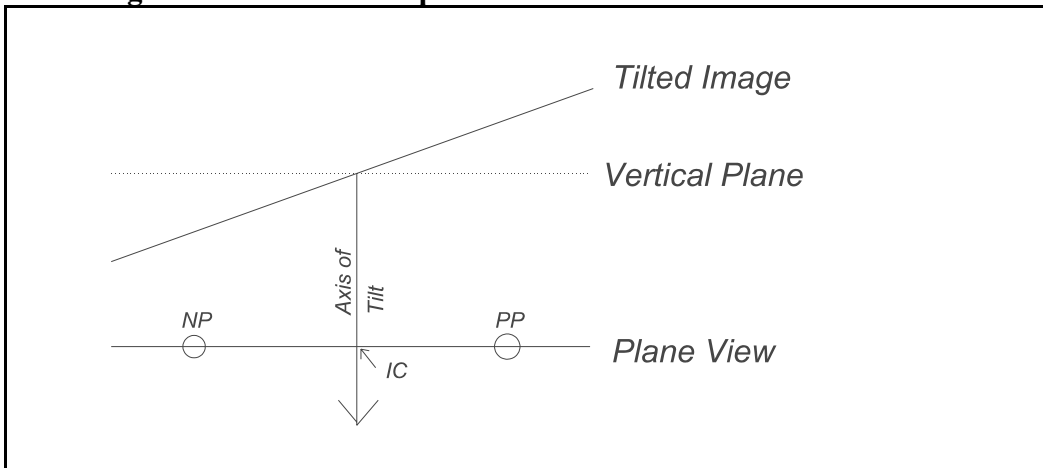
### 3. Relationship between focal length and flying height



#### 4. Relationship of Tip and Tilt on an aerial image



#### 4a. Change in scale on a tilted photo



### **Ground Distance:**

Equal Elevation - Recall that ground distance for all points at the same elevation is

$$RF = \frac{1}{S} = \frac{d}{D} = \frac{f}{(H-h)}$$

$$\text{thus } D = d \frac{(H-h)}{f}$$

Note that  $d$  and  $f$  are in the same units of measure (i.e. inches) and that the unit of measure for  $D$  will be the same as the unit of measure for  $H$  and  $h$ . This assumes that all points along the ground distance  $D$  are at the same elevation.

End Points at Different Elevations - If the end points of a ground distance (line) are at different elevations, you could compute the ground distance ( $D$ ) using:

1. Average Scale (i.e. elevation) along the line, or
2. End Point Scales (i.e. elevations) of the line.

For example, if  $f=6$  inches,  $H=6,500$  ft., the elevation at endpoint  $a$  was  $500$  ft. (i.e.  $h_a=500$ ) and the elevation at endpoint  $b$  was  $1,000$  ft. (i.e.  $h_b=1,000$ ); then.....

### Average Scale - Approximation

The average elevation  $h_{ave}$  would be:

$$h_{ave} = (h_a + h_b) / 2 = 750 \text{ ft.}$$

and the average scale would be

$$1/S = .5/(6,500-750)$$

$$1/S = 1/11,500 \text{ or,}$$

$$PES = (6,500-750)/6" = 958.33 \text{ ft./inch}$$

$$1 \text{ inch on the photo} = 958.33 \text{ ft. on the ground.}$$

Thus, if photo  $d=2.81$  inches then ground distance  $D=2,692.91$  ft. using the average scale.

NOTE: This is only an approximation because the scale is not weighed proportionally to the displacement position on the photograph. This is a reasonable method for quick approximations, but it will not yield the correct scale because topographic displacement is radial from the PP and varies with distance.



**Formula:**

$$RF = \frac{1}{S} = \frac{d}{D} = \frac{f}{(H-h_0)}$$

**Laboratory Procedures:**

1. Arrange photos 28159 178-180 and AVE 2KK-78, and the Betheden quad sheet in order of large to small scale. Hint: Large number vs. small number; 1/2 vs. 1/10; or largest d over D fraction... i.e. find same distance on each image and compare.
2. Carefully locate and identify Points 1-9 (or as many as occur) on the Betheden sketch map, Betheden 7.5' quad sheet, and the appropriate aerial photos. Hint: Look for road patterns around Bevil Hill Church (located along Bluff Lake Road).
3. To determine the average scale of the photos, select at least 3 long diagonal lines, carefully measure the lengths on both the photo and the quad sheet using the 60<sup>th</sup> scale on the engineer scale. Note, the line endpoints do not have to correspond to the 9 points on the sketch map; other road intersections may be used if they can be identified on both the photo and the quad sheet. Always use the intersection of road centerlines, or features that do not change. Do not use intersections of creeks and streams!
4. Determine the average scale of photos AVE-2KK-91 and 28159 178-174. Use the weighted sum approach i.e.  $\text{Sum}(d_i)/\text{Sum}(D_i)$  rather than averaging the computed RF from each photo.
5. What was the approximate aircraft (platform) altitude when the photo AVE-2KK-91 was taken? Use the above RF of photo AVE-2KK-91 from question 7 above with a focal length of 8.25 inches and average ground level (agl) of 350 ft.
6. Calculate the approximate horizontal ground distance (D) in feet of specified lines using the endpoint scale from photo measurements, only. Assume **no topo/quad** map is available but the following elevations and camera information are known:

$$H = 20,230 \text{ ft. and } f = 6.00 \text{ inches}$$

$$h_a = \text{average elevation of points } h_i \text{ and } h_j$$

$$h_1 = 565 \text{ ft. Betheden road intersection, Point 1}$$

$$h_5 = 350 \text{ ft. Point 5}$$

$$h_2 = 580 \text{ ft. Bevil Hill road intersection, Point 2}$$

$$h_6 = 368 \text{ ft. Point 6}$$

$$h_3 = 330 \text{ ft. Barton's sawmill intersection, Point 3}$$

$$h_7 = 557 \text{ ft. Point 7}$$

7. Determine the average scale of photos NAPP 3986-210 and USFWS 196-58. Use the weighted sum approach i.e.  $\text{Sum}(d_i)/\text{Sum}(D_i)$  rather than averaging the computed RF from each photo.

**Answer Sheet For Laboratory 2A**

(40 points)

Date \_\_\_\_\_

Name \_\_\_\_\_

Name \_\_\_\_\_

1. Rank the following images in terms of scale by placing a S, M, L for smallest, medium, and largest scale: (3)

photo 28159 178-180 \_\_\_\_\_  
 photo AVE 2KK-78 \_\_\_\_\_  
 Betheden 7.5' quad sheet \_\_\_\_\_

2. Which aerial photos are covered by the Betheden 7.5' Quad? Mark Y/N. (4)

NAPP 3986-210	_____	USFWS 196-59	_____
28159 178-174	_____	28159 178-180	_____
AVE 2KK-78	_____	AVE 2KK-91	_____

3. What is the reason for selecting long diagonal lines for measurement of average scale? (2)
- \_\_\_\_\_
- \_\_\_\_\_

4. Determine the average scale (RF) of photos 28159 178-174 and AVE-2KK-91. Recall that the scale of the 7.5' Quad sheet is 1:24,000 or 1 inch = 2,000 ft and all image measurements are in 60ths of an inch.

**28159 178-174:**

<b>Photo <math>d_i</math></b>	<b>Quad <math>D_i'</math></b>	
_____	_____	
_____	_____	
_____	_____	
$\sum d_i =$ _____	$\sum D_i' * 2,000 =$ _____	RF = _____ (3)

**AVE - 2KK- 91:**

<b>Photo <math>d_i</math></b>	<b>Quad <math>D_i'</math></b>	
_____	_____	
_____	_____	
_____	_____	
$\sum d_i =$ _____	$\sum D_i' * 2,000 =$ _____	RF = _____ (3)

5. What was the approximate aircraft (platform) altitude when the photo AVE - 2KK- 91 was taken? Use the above RF of photo AVE - 2KK- 91 from question 7 above with a focal length of 8.25 inches and average ground level (agl) of 350 ft.

AVE - 2KK- 91 Platform Altitude \_\_\_\_\_ ft. (3)

6. Calculate the approximate horizontal ground distance (D) in feet of the following lines using the endpoint scale from photo measurements, only. Assume **no topo/quad** map is available but the following elevations and camera information are known:

H = 20,230 ft. and f=6.00 inches     $h_a$  = average elevation of points  $h_i$  and  $h_j$

$h_1$ = 565 ft. Betheden road intersection, Point 1                       $h_5$ = 350 ft. Point 5  
 $h_2$ = 580 ft. Bevil Hill road intersection, Point 2                       $h_6$ = 368 ft. Point 6  
 $h_3$ = 330 ft. Barton's sawmill intersection, Point 3                       $h_7$ = 557 ft. Point 7

<u>Photo</u>	<u>Line Segment</u>	<u>...<math>h_a</math>....</u> (feet)	<u>.....RF.....</u> (unitless)	<u>.....<math>d_i</math>....</u> (inches)	<u>.....<math>D_i</math>.....(4)</u> (feet)
#174	Point 1 - 2	_____	_____	_____	_____
#174	Point 1 - 7	_____	_____	_____	_____
#180	Point 2 - 6	_____	_____	_____	_____
#180	Point 2 - 3	_____	_____	_____	_____

7. Determine the average scale of photos NAPP 3986-209/210 and USFWS 196-58/59. Use the weighted sum approach i.e.  $\text{Sum}(d_i)/\text{Sum}(D_i)$  rather than averaging the computed RF from each photo.

**NAPP 3986-210:**

	<b>Photo <math>d_i</math></b>	<b>Quad <math>D_i'</math></b>	
	_____	_____	
	_____	_____	
	_____	_____	
	$\sum d_i =$ _____	$\sum D_i * 2,000 =$ _____	<b>RF =</b> _____ (3)

**USFWS 196-59:**

	<b>Photo <math>d_i</math></b>	<b>Quad <math>D_i'</math></b>	
	_____	_____	
	_____	_____	
	_____	_____	
	$\sum d_i =$ _____	$\sum D_i * 2,000 =$ _____	<b>RF =</b> _____ (3)

**Answer Sheet For Laboratory 2A**

(40 points)

Date \_\_\_\_\_

Name \_\_\_\_\_

Name \_\_\_\_\_

1. Rank the following images in terms of scale by placing a S, M, L for smallest, medium, and largest scale: (3)

photo 28159 178-180 \_\_\_\_\_  
 photo AVE 2KK-78 \_\_\_\_\_  
 Betheden 7.5' quad sheet \_\_\_\_\_

2. Which aerial photos are covered by the Betheden 7.5' Quad? Mark Y/N. (4)

NAPP 3986-210	_____	USFWS 196-59	_____
28159 178-174	_____	28159 178-180	_____
AVE 2KK-78	_____	AVE 2KK-91	_____

3. What is the reason for selecting long diagonal lines for measurement of average scale? (2)

\_\_\_\_\_  
 \_\_\_\_\_

4. Determine the average scale (RF) of photos 28159 178-174 and AVE-2KK-91. Recall that the scale of the 7.5' Quad sheet is 1:24,000 or 1 inch = 2,000 ft and all image measurements are in 60ths of an inch.

**28159 178-174:**

<b>Photo <math>d_i</math></b>	<b>Quad <math>D_i'</math></b>	
_____	_____	
_____	_____	
_____	_____	
$\sum d_i =$ _____	$\sum D_i' * 2,000 =$ _____	<b>RF =</b> _____ (3)

**AVE - 2KK- 91:**

<b>Photo <math>d_i</math></b>	<b>Quad <math>D_i'</math></b>	
_____	_____	
_____	_____	
_____	_____	
$\sum d_i =$ _____	$\sum D_i' * 2,000 =$ _____	<b>RF =</b> _____ (3)

5. What was the approximate aircraft (platform) altitude when the photo AVE - 2KK- 91 was taken? Use the above RF of photo AVE - 2KK- 91 from question 7 above with a focal length of 8.25 inches and average ground level (agl) of 350 ft.

AVE - 2KK- 91 Platform Altitude \_\_\_\_\_ ft. (3)

6. Calculate the approximate horizontal ground distance (D) in feet of the following lines using the endpoint scale from photo measurements, only. Assume **no topo/quad** map is available but the following elevations and camera information are known:

H = 20,230 ft. and f=6.00 inches     $h_a$  = average elevation of points  $h_i$  and  $h_j$

$h_1$ = 565 ft. Betheden road intersection, Point 1                       $h_5$ = 350 ft. Point 5  
 $h_2$ = 580 ft. Bevil Hill road intersection, Point 2                       $h_6$ = 368 ft. Point 6  
 $h_3$ = 330 ft. Barton's sawmill intersection, Point 3                       $h_7$ = 557 ft. Point 7

<u>Photo</u>	<u>Line Segment</u>	<u>...<math>h_a</math>....</u> (feet)	<u>.....RF.....</u> (unitless)	<u>.....<math>d_i</math>....</u> (inches)	<u>.....<math>D_i</math>.....(4)</u> (feet)
#174	Point 1 - 2	_____	_____	_____	_____
#174	Point 1 - 7	_____	_____	_____	_____
#180	Point 2 - 6	_____	_____	_____	_____
#180	Point 2 - 3	_____	_____	_____	_____

7. Determine the average scale of photos NAPP 3986-209/210 and USFWS 196-58/59. Use the weighted sum approach i.e.  $\text{Sum}(d_i)/\text{Sum}(D_i)$  rather than averaging the computed RF from each photo.

**NAPP 3986-210:**

	<b>Photo <math>d_i</math></b>	<b>Quad <math>D_i'</math></b>	
	_____	_____	
	_____	_____	
	_____	_____	
	$\sum d_i =$ _____	$\sum D_i * 2,000 =$ _____	RF= _____ (3)

**USFWS 196-59:**

	<b>Photo <math>d_i</math></b>	<b>Quad <math>D_i'</math></b>	
	_____	_____	
	_____	_____	
	_____	_____	
	$\sum d_i =$ _____	$\sum D_i * 2,000 =$ _____	RF= _____ (3)